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ABSTRACT

Unified Technical Concepts (UTC) is a modular system for teaching applied physics in two-year postsecondary programs. This UTC classroom textbook, consisting of 14 chapters, deals with physics for technicians. Addressed in the individual chapters of the guide are the following topics: force, work, rate, momentum, resistance, power, potential and kinetic energy, force transformers, energy converters, transducers, vibrations and waves, time constants, and radiation. Each chapter contains an instructional text, a summary, one or more demonstration activities, a self-check, and a list of references. (Appended to this textbook are the Greek alphabet, selected physical constants, selected astronomical data, and tables of conversion factors.) (MN)

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ED239049

UNIFIED TECHNICAL CONCEPTS

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PHYSICS FOR TECHNICIANS

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PREFACE

Skilled technicians are a national resource. They are more in demand today than ever before. Sometimes referred to as "super-techs," they must calibrate, adapt, and maintain a vast array of increasingly sophisticated and complex modern industrial equipment. They must understand the how and why of basic electronic, mechanical, hydraulic, thermal, and optical devices. In addition, they must know how these devices are assembled to work together in systems as intricate as robots, computers, photocopiers, and laser communication networks. To meet these challenges, today's technician must have a broad understanding of technical principles in electrical, mechanical, hydraulic, pneumatic, thermal, and electromechanical systems. Such an understanding begins logically with a sound course in applied technical physics.

Unified Technical Concepts in Physics--or UTC Physics for short--is a course of instruction for technicians at the postsecondary level. It is a carefully selected and organized assembly of text and laboratory course materials. It is intended to organize technical principles in such a manner as to make them readily understood and applicable in different technologies--those that include electrical, mechanical, fluidal, and thermal systems, and combinations thereof. It is intended to blend useful technical principles with laboratory practice on realistic devices that technicians use in their everyday work. It is NOT intended for students who plan to pursue physics as a career, or those who intend to become scientists or engineers. For those students, the traditional texts and courses in physics will continue to serve their needs.

The traditional format for presentation of technical physics has long been the bound textbook, logically divided into chapters that address, in turn, the separate disciplines of mechanics, heat, sound, wave motion, electricity, magnetism, and optics. But for technicians who generally take only one course in physics--as an important foundation course--this format has several inherent weaknesses. First, while mastering physics in the usual "compartments"--mechanics, heat, sound, etc--the technician may miss important concepts that have analogous relationships that thread through the separate disciplines of mechanics, heat, hydraulics, and electricity. For example, the student may identify the physical concept of rate with speed (length/time), as it is usually introduced in mechanics, and not identify the rate analogies calories/sec in thermal systems, ft^3/sec in hydraulic systems, and coulombs/sec in electrical systems. Second, while spending undue time on formula derivation and proofs--a practice considered essential in traditional physics courses--the technician may miss the underlying message contained in important application of technical principles to useful devices and instruments in the scientific and industrial world.

In content, UTC Physics is similar to a traditional course in technical physics. Each course covers the body of knowledge called physics. But, in format, organization and manner of presentation, UTC Physics is significantly different. UTC Physics is a modular system of instruction. It consists of *concept* modules, *application* modules, and *supplementary* modules. The thrust of presentation in the concept modules is strongly one of unification of basic physics concepts in four major disciplines--mechanical, fluidal, thermal, and electrical. And most importantly, through the use of specially designed laboratory experiments in application modules, hands-on experience for the technician is heavily emphasized.

The contents of this book include the textual material--the *concept* modules. There are thirteen in all, each identified as an important technical base needed by today's technician. In order of presentation the technical concepts are: FORCE, WORK, RATE, MOMENTUM, RESISTANCE, POWER, ENERGY, FORCE TRANSFORMERS, ENERGY CONVERTORS, TRANSDUCERS, VIBRATIONS AND WAVES, TIME CONSTANTS, and RADIATION. While the first seven concepts are easily recognized as basic to physics, the balance, especially ENERGY CONVERTORS, TRANSDUCERS, and TIME CONSTANTS are not generally given headline coverage in traditional physics courses. But we have found them to be important for technicians.

We strongly advocate one year of UTC physics for the technician, with careful coverage of all thirteen concept modules, complemented by lab experiences from 60 to 90 application modules. However, for programs where only one-half of a year of physics--rather than the traditional full year--is available, we recommend that a total of seven concept modules and 40 to 60 application modules be covered. The concepts would include, in order: FORCE, WORK, RATE, RESISTANCE, POWER, ENERGY, and FORCE TRANSFORMERS.

Each of the thirteen concept modules begins with a development of the basic physics relevant to the physics concept in question. Then, the basic physics concept is identified as a unifying principle that is extended to a variety of problems in mechanical, thermal, fluidal, and electrical systems. For example, the concept module on FORCE first covers the basic idea of a "force." It begins with the traditional definition of a "push or a pull" and moves on to applications involving Newton's laws of motion. But, in addition, the idea of force as a unifying concept is introduced and extended to thermal systems, hydraulic systems, and electrical systems. The module shows, for example, that a temperature difference (ΔT), a pressure difference (ΔP), and a voltage difference (ΔV) all behave like forces, even though these quantities are not forces in the strict sense of a "push or a pull"; nor do they have the usual units of a force, e.g., newtons. In this manner, the concept of FORCE is extended beyond the simple idea of a push or a pull in mechanical systems, and shown to be a useful, unifying concept that aids the student in understanding forces and motion in thermal, hydraulic, and electrical systems. This general pattern of defining and explaining each concept--and then demonstrating its application as a unifying concept in mechanical, fluidal, thermal, and electrical systems--is continued throughout the text for each of the remaining 12 concepts.

While not a part of this textbook, there are 170 laboratory exercises available. These are the *application* modules. They have been appropriately designed in the fields of mechanics, fluids, heat, vibrations, electricity, and optics to illustrate real-world applications of the concept modules. They are available separately, or in any combination, depending on the needs of the particular school or program. Each application module describes an important application of a physical principle to a common piece of equipment. For example, to illustrate an important application of a technical principle, application modules use shock absorbers, prony brakes, strain gages, venturi meters, and electric motors in place of the traditional vibrating springs, inclined planes, pith balls, and air tables. The modules are carefully designed to emphasize hands-on laboratory experience. While not yet available in text form, it is CORD's intent to prepare two bound volumes of selected laboratory application modules. The two volumes, representing an appropriate selection from the 170 application modules developed, will be available in August 1983.

As a remedial resource, *supplementary* modules have been developed and are available for students who may have a deficiency in elementary mathematics or laboratory skills. These modules are also available for self-study or rapid review with instructor assistance. In addition a manual providing solutions for self checks in the supplementary, concept, and application modules is available.

In summary, UTC Physics is a modular system of instruction designed for technicians at the postsecondary level. It is built around basic technical concepts that form the foundation of the course. It offers a wide choice of application modules that exploit "hands-on" learning to apply the basic principles of physics to specialized areas of modern technology.

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ACKNOWLEDGMENTS

Unified Technical Concepts (UTC)--Physics for Technicians is a major innovation in technical education for the 1980s. UTC physics is innovative because it provides a broad, interdisciplinary base for teaching technical principles, combined with the practical illustration of these principles in the work clothes of modern industrial equipment.

The evolution of this delivery system is due largely to the contribution of several outstanding technical educators: First, to Dr. Maurice W. Roney, for his conceptualization of unified physics while developing an electromechanical technology curriculum in the late 1960s. Next, to Dr. Walter J. Brooking for his encouragement to persist in the development and dissemination of the UTC curriculum. Finally, to Dr. Leno S. Pedrotti the chief editor, technical consultant and designer of the instructional format. Significant contributions to UTC were also made by CORD staff members Johnny Jones and Carl Lansford, and by project editor Dr. Frank L. Pedrotti.

Daniel M. Hull, UTC Project Director
President, Center for Occupational
Research and Development
June 1983

FORCE

CHAPTER 1-0

The concept of force is based upon the common experiences of every person. A force is a push or pull that may cause an object to move. Force — a physical quantity that causes movement — is the "prime mover" in the mechanical energy system.

This module discusses the concept of force and forcelike quantities as they apply to four energy systems — mechanical, fluidal, electrical and thermal. The forcelike quantities discussed are force, torque, pressure, voltage and temperature difference.

- Force causes motion of an object in a translational mechanical system. This force serves as the model for all forcelike quantities.
- Torque causes rotation of an object in a rotational mechanical system. Torque is a forcelike quantity.
- Pressure causes a fluid to move in a fluidal system. Pressure is a forcelike quantity.
- Voltage causes electrical charge to move in an electrical system. Voltage is a forcelike quantity.
- Temperature difference causes heat energy to move in a thermal system. Temperature difference is a forcelike quantity.

The Discussion includes the mathematical formulation and units used to describe each forcelike quantity in each energy system and stresses the characteristics of forcelike quantities that apply to all energy systems.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following physical quantities and, where applicable, state their units in both SI (International System of Units) and English System of Units.

  Force.
  Torque.
  Pressure.
  Voltage.
  Temperature difference.

- Given two or more mechanical forces acting along the same line, determine the resultant force.

- Given two of the following quantities in a mechanical rotational system, determine the third:

  Force.
  Lever arm.
  Torque.

- Given two of the following quantities in a fluid system, determine the third:

  Force.
  Area.
  Pressure.

- Given two of the following quantities in a fluid system, determine the third:

  Pressure.
  Height of fluid.
  Weight density.

- Given two or more voltage sources connected in series, determine the resultant voltage.

- Given a temperature in either degrees Celsius or degrees Fahrenheit, determine the equivalent temperature on the other scale.

- Describe how pressure in fluidal systems, voltage in electrical systems and temperature difference in thermal systems are similar to force and torque in mechanical systems.

- Describe the conditions which must be met for equilibrium in each of the following energy systems.

  Mechanical.
  Fluidal.
  Electrical.
  Thermal.

MECHANICAL SYSTEMS

FORCE

A linear mechanical force is a push or pull applied along a straight line, and usually is represented as an arrow indicating the direction of the applied force. Figure 1 illustrates the result when a single force is applied to an object at rest. The force causes the object to move in the direction of the applied force. If the object is already moving when a force is applied, the force may cause it to slow down, speed up, or change direction, depending upon the direction of the applied force with respect to the original direction of motion of the object.

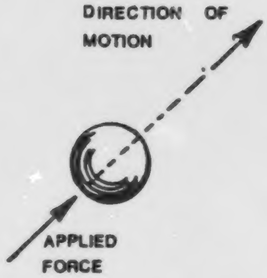


FIG. 1

In the physical world, complex systems of forces must be dealt with precisely. Figure 2 shows two forces applied to an object and a vector diagram of the forces. A vector is an arrow whose direction is the direction of the force it represents and whose length is proportional to the magnitude (strength) of the force. The resultant force is the vector sum of all individual forces. Techniques for adding forces in several directions will be discussed in later modules. The remainder of this discussion presents the basic characteristics of force, using simplified examples in which all forces act along a single line.

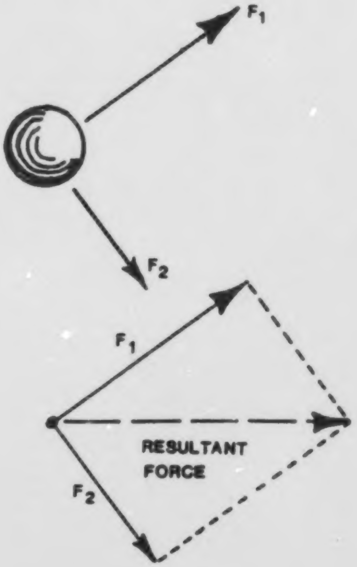


FIG. 2

The presence of forces acting upon an object does not mean necessarily that there will be a change in the state of motion of that object. A change in motion results only when the resultant force (also called the "net force," or the "differential force," and often written as ΔF) is not zero. If the net force is zero, the object remains in its original state of motion; that is, objects at rest remain at rest and objects in motion remain in motion in a straight line at a constant speed. This condition is called equilibrium.

In Figure 3 a skydiver has jumped from a plane. He falls toward the earth at an increasing speed because of the force of gravity exerted upon his body by the earth. He eventually reaches a steady maximum speed called

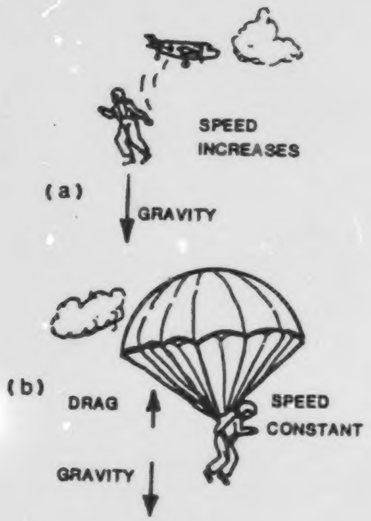


FIG. 3

"terminal velocity," which occurs when the upward resistance force of the air (drag) is equal to the downward pull of gravity (weight). The skydiver then continues to fall at a constant velocity in a state of equilibrium. When the parachute opens, the much larger area of the parachute causes a greater upward force due to increased wind resistance. This force slows the skydiver until a new equilibrium is reached at a lower speed and the skydiver lands safely.

The desirability often arises for finding the resultant force when several forces act within a system. For systems in which all forces lie along the same line this calculation is accomplished by assigning arbitrarily one direction along that line to be positive and the other to be negative, and then by adding all the forces as indicated by Equation 1.

$$F_T = F_1 + F_2 + F_3 + \dots \qquad (1)$$

where

F_T = total force (resultant).

F_1, F_2, F_3... are individual forces acting on the system.

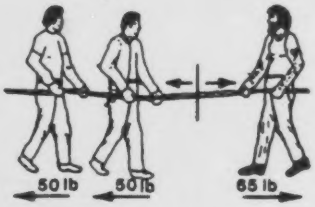
Example A illustrates the use of this equation in solving a simple problem.

EXAMPLE A : TUG-OF-WAR PROBLEM

Given: Three people are playing tug-of-war. Two people are on one side and each is pulling with a force of fifty pounds. The person on the other side is pulling with a force of sixty-five pounds.

Find: The net force and the initial direction of motion.

Solution: Define right to be positive and left to be negative. The forces acting on the rope are:



-50 lb
-50 lb
+65 lb

Equation I:
$$\begin{aligned} F_T &= F_1 + F_2 + F_3 \\ &= (-50\ \text{lb}) + (-50\ \text{lb}) + (65\ \text{lb}) \\ &= -\ 100 + 65\ \text{lb} \\ F_T &= -35\ \text{lb} \end{aligned}$$

The net force is 35 lb to the left, and the motion will be in that direction.

UNITS OF MASS AND FORCE

Mass is a measure of the quantity of matter present in an object and is a characteristic of that object regardless of its environment. Weight is the force of gravitational attraction of an object to the earth and changes as the distance from the earth changes.

The English system of units is based upon length in feet, time in seconds and force in pounds. The weight of an object in pounds (lb) is the gravitational force exerted upon that object by the earth. The English unit of mass is the slug, a derived unit defined as the amount of mass that will experience a change in speed of one foot per second when a one-pound force is applied to it for one second. The slug is not a common unit; it is defined here only to aid in the comparison of English and SI units. When English units are used, an object's weight (at the earth's surface) usually is specified in pounds.

International System of Units (SI) is based upon length in meters, time in seconds and mass in kilograms. The SI unit of force is a derived unit called a Newton (N). A Newton is defined as the amount of force that will change the speed of a one-kilogram mass by one meter per second if applied for one second. Since weight is a force, weight is measured in the SI system in Newtons. When SI units are used, an object's mass usually is specified in kilograms.

Table I shows the relationships of the units of length, time, mass and force in the SI and English systems.

TABLE 1. SI AND ENGLISH UNITS

	Length	Time	Mass	Force
SI System of Units	meter m	second s	kilogram kg	Newton * $N = \frac{kg \cdot m}{s^2}$
English System of Units	foot ft	second s	slug (no symbol) * $\frac{lb \cdot s^2}{ft}$	pound lb

* Derived units

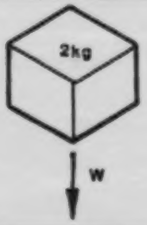
The weight of an object is the product of its mass and the acceleration of gravity as given in Equation 2.

$$W = mg \qquad (2)$$

where
W = weight (a force in N or lb).
m = mass (in kg or slugs).
g = acceleration of gravity (SI: $9.8\ m/s^2$; English: $32\ ft/s^2$).

The acceleration of gravity is the acceleration that an object will experience due to the earth's attraction if it is released near the earth's surface and if no other forces act on it. An acceleration of $9.8\ m/s^2$ means that the velocity of the object will increase by 9.8 m/s during each second it falls. Example B illustrates the use of this equation in determining the weight of an object if the mass is known.

EXAMPLE B: CALCULATION OF WEIGHT



Given: A brick with a mass of 2 kg.

Find: Weight of the brick.

Solution:
$W = mg$
$= (2\ kg)(9.8\ m/s^2)$
$- 19.6\ kg \cdot m/s^2$
$W = 19.6\ N$

Table 2 lists some conversion factors that are useful when working with weight and mass.

TABLE 2. WEIGHT AND MASS CONVERSION FACTORS

1 lb (force) = 4.45 N (force)
1 slug (mass) = 7.23 kg (mass)
1 kg weighs 9.8 N or 2.2 lb
1 slug weighs 32.2 lb or 143 N

FORCELIKE QUANTITIES

Force is the "prime mover" in a translational mechanical system. Any change in the state of motion of an object occurs because of forces acting upon that object. A force causes the displacement of a mass through a distance.

"Forcelike quantities" are those physical quantities in other energy systems (mechanical rotational, fluidal, electrical, thermal) that act as the prime movers in those systems. In any energy system, an unbalanced forcelike quantity causes a change (a displacement of some quantity) in that system. Although these forcelike quantities do not act in their respective systems in exactly the same way that force acts in a translational mechanical system, all have certain characteristics in common with force. Force was chosen as the first prime mover to be discussed because it is the most basic. Other forcelike quantities are actually the results of forces acting within a system. The remainder of this module discusses forcelike quantities in other energy systems and begins a unification of physics by pointing out analogies that relate these quantities to the more familiar and basic concept of force.

TORQUE

Another mechanical system of interest is the mechanical rotation system. The prime mover, or forcelike quantity, in this system is torque. Torque causes a change in the state of rotational motion about an axis, just as force causes a change in the state of linear motion. Torque results when a force is applied at some distance from an axis of rotation, as shown in Figure 4. The perpendicular distance from the line of action of the

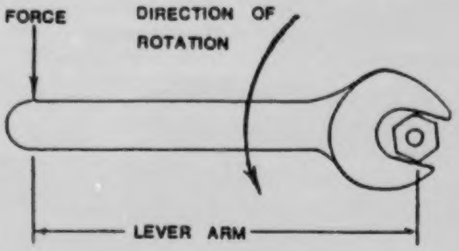


FIG. 4

force to the axis is called the "lever arm," or the "moment arm," of the force. Torque is defined by Equation 3.

$$\tau = F\ell \tag{3}$$

where

τ = torque (ft•lb or N•m).
F = force (lb or N).
ℓ = lever arm (ft or m).

The use of this equation in determining torque is illustrated in Example C.

EXAMPLE C : CALCULATION OF TORQUE

Given: A force of 5 lb (22.25 N) acts with a lever arm of 2 ft (0.61 m).

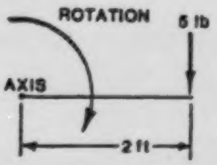
Find: Torque.

Solution: English

$\tau = F\ell$
$= (5\ \text{lb})(2\ \text{ft})$
$\tau = 10\ \text{ft}\cdot\text{lb}$

SI

$\tau = F\ell$
$= (22.25\ \text{N})(0.61\ \text{m})$
$\tau = 13.57\ \text{N}\cdot\text{m}$



Just as the direction of a force must be specified, the direction of the rotation resulting from a torque must be specified. One method of specifying torques is as clockwise (cw) and counterclockwise (ccw). When solving problems involving the total torque in a system one of these directions is chosen as positive, and the other negative and the torques are added algebraically, as indicated in Equation 4.

$$\tau_T = \tau_1 + \tau_2 + \tau_3 + \ldots \tag{4}$$

where

τ_T = total torque.

τ_2, τ_3, τ_4 ... are individual torques acting on the system.

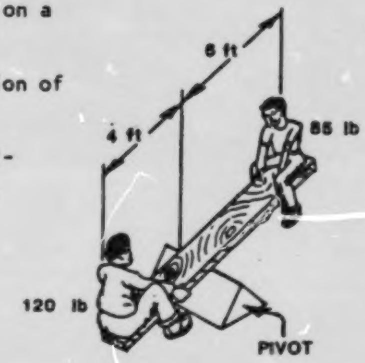
Examples D and E show the use of this equation and of Equation 3 in solving torque problems.

EXAMPLE D: BEAM PROBLEM

Given: Two people are sitting on a beam as shown.

Find: Total torque and direction of rotation.

Solution: Define clockwise as positive. Torque of person on right:



$$\begin{aligned} \tau_R &= F_R \ell_R \\ &= (85 \text{ lb})(6 \text{ ft}) \text{ cw} \\ &= 510 \text{ ft·lb cw} \\ \tau_R &= 510 \text{ ft·lb} \end{aligned}$$

Torque of person on left:

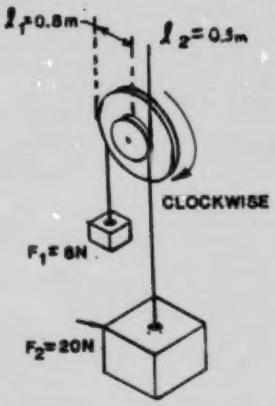
$$\begin{aligned} \tau_L &= F_L \ell_L \\ &= (120 \text{ lb})(4 \text{ ft}) \text{ ccw} \\ &= 480 \text{ ft·lb ccw} \\ \tau_L &= -480 \text{ ft·lb} \end{aligned}$$

Total torque:

$$\begin{aligned} \tau_T &= \tau_R + \tau_L \\ &= 510 \text{ ft·lb} + (-480 \text{ ft·lb}) \\ \tau_T &= 30 \text{ ft lb·cw} \end{aligned}$$

The beam will rotate in a clockwise direction, with a torque of 30 ft·lb. The 120 lb person will be lifted, the 85 lb person lowered.

EXAMPLE E: PULLEY SYSTEM PROBLEM



Given: A pulley system has weights attached to provide forces as shown.

Find: Total torque and direction of rotation.

Solution: Define clockwise as positive:

$$\begin{aligned}
\tau_1 &= F\,\ell \\
\tau_1\ (\text{ccw}) &= -F_1\,\ell_1 \\
&= -(8\ \text{N})(0.8\ \text{m}) \\
\tau_1 &= -6.4\ \text{N}\cdot\text{m} \\
\tau_2\ (\text{cw}) &= F_2\ell_2 \\
&= (20\ \text{N})(0.3\ \text{m}) \\
\tau_2 &= 6.0\ \text{N}\cdot\text{m} \\
\tau_T &= \tau_1 + \tau_2 \\
&= -6.4\ \text{N}\cdot\text{m} + 6.0\ \text{N}\cdot\text{m} \\
\tau_T &= -0.4\ \text{N}\cdot\text{m}
\end{aligned}$$

The pulley system will turn in a counterclockwise direction with a torque of 0.4 N•m.

If the total torque on any system is zero, that system is in rotational equilibrium and its rotational rate remains constant.

FLUIDAL SYSTEMS

The prime mover, or forcelike quantity, in fluidal systems (hydraulic or pneumatic) is called "pressure." Pressure is defined as the force per unit area exerted by a fluid and is given by Equation 5.

$$p = \frac{F}{A} \tag{5}$$

where

p = pressure (lb/in^2 or N/m^2).
F = force (lb or N).
A = area (in^2 or m^2).

Examples F and G illustrate the use of Equation 5 in solving two types of problems.

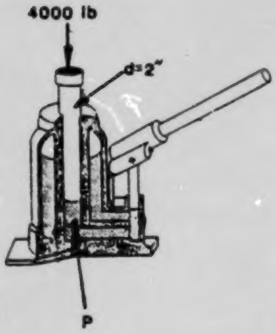
EXAMPLE F: FLUID PRESSURE IN A HYDRAULIC JACK

Given: A hydraulic jack is rated at 4000 lb lifting capacity and has a large piston with a diameter of 2 inches.

Find: Fluid pressure in the jack at maximum load.

Solution: Area:

$$A = \pi r^2$$
$$= (3.14)(1\ \text{in})^2$$
$$A = 3.14\ \text{in}^2$$
$$p = \frac{F}{A}$$
$$= (4000\ \text{lb}) \div (3.14\ \text{in}^2)$$
$$p = 1274\ \text{lb/in}^2$$



EXAMPLE G: FORCE ON AIRPLANE WINDOW

Given: An airplane window with surface area of 2 square feet. Pressure inside the cabin is 8 pounds per square inch greater than the pressure outside.

Find: The net force on the window.

Solution:

$$p = \frac{F}{A}$$
$$F = pA$$
$$F = (8\ \frac{\text{lb}}{\text{in}^2})(288\ \text{in}^2) \qquad 1\ \text{ft}^2 = 144\ \text{in}^2$$
$$\text{Force} = 2304\ \text{lb (outward)}$$

The window must withstand a net force of 2304 lb.

Pressure of a gas in a container depends upon the number and speed of molecules coliding against the sides of the container. Therefore, one way to increase the pressure on a gas in a container is either to pump more molecules into the container or to force the same molecules into a small container. To decrease the pressure, some of the molecules can be released from the container or placed into a larger container. Another way to increase pressure is to heat a fluid inside a closed container. This procedure causes the molecules to increase their speed, creating an increased pressure. Cooling of the container has the reverse effect on the molecules and, consequently, decreases the pressure.

Pressure in a liquid increases as the depth increases. For instance, if water in a tank is made deeper, there will be more water above the location at which pressure is measured; therefore, the weight of the additional water will create a greater pressure at the deeper position. The pressure at any depth in a container filled with a liquid depends upon the height of the liquid above that point, the density of the liquid and the acceleration of gravity, as given in Equation 6.

$$p = \rho g h \qquad (6)$$

where

p = pressure (N/m^2).
ρ = density (kg/m^3).
h = height (m).

In the English system of units this is usually written in the form of Equation 7.

$$p = \rho_w h \qquad (7)$$

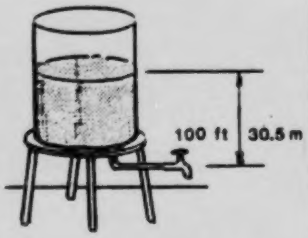
where

ρ_w = weight density in lb/ft^3.
h = height in feet.

Weight density refers to the weight (gravitational force) of a volume of liquid, and contains the gravitational constant as a factor. Example H illustrates the use of Equations 6 and 7 in solving a problem.

EXAMPLE H: WATER PRESSURE PROBLEM

Given: The height of the water in a water system is 100 ft (30.5 m) above a faucet. The density of water (SI units) is 1000 kg/m^3. The weight density of water (English units) is 62.4 lb/ft^3.



Find: The pressure at the faucet in SI and English units.

Solution: (a) SI Units: $p = \rho g h$
$= (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(30.5 \text{ m})$

$$p = 2.99 \times 10^5\ \text{kg/m}\cdot\text{s}^2$$
$$p = 2.99 \times 10^5\ \text{N/m}^2$$

Note that $\frac{N}{m^2} = \frac{\left(\frac{kg\cdot m}{s^2}\right)}{m^2} = \frac{kg}{m\cdot s^2}$

The units of pressure are force per area.

(b) English Units: $p = \rho_w h$

$$\begin{aligned} &= (62.4\ \text{lb/ft}^2)(100\ \text{ft}) \\ &= 6240\ \text{lb/ft}^2 \\ &= (6240\ \text{lb/ft}^2)\left(\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right) \\ p &= 43.3\ \text{lb/in}^2 \end{aligned}$$

The units have been converted to the more familiar pounds per square inch. This is also force per unit area.

Motion occurs in a fluid as the result of a pressure difference. For this reason, pressure plays the role of the forcelike quantity in a fluidal system. Figure 5 shows two tanks with different liquid levels connected by a pipe with a valve. The pressures p_1 and p_2 at the ends of the pipe depend upon the height of liquid above that point in each tank. Since the liquid in tank 2 is higher, p_2 is greater than p_1. When the value is opened, this pressure differential will cause liquid to flow through the pipe until the liquid levels are the same in both tanks. As this point, p_1 and p_2 will be equal and the system will be in equilibrium. Equilibrium depends only upon the heights being equal, not upon the volume of liquid in the tanks. The system shown in Figure 6 is in equilibrium, although the right tank contains more liquid.

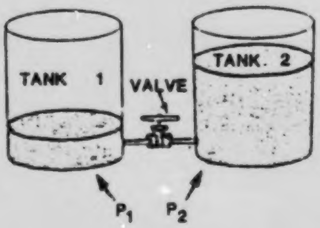


FIG. 5

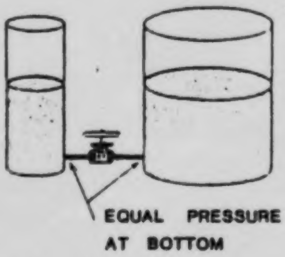


FIG. 6

ELECTRICAL SYSTEMS

The prime mover, or forcelike quantity, in electrical systems is the voltage, sometimes referred to as "potential difference" or "electromotive force." Voltage occurs in an electrical system because of the

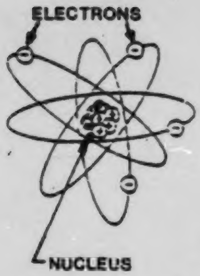


FIG. 7

separation of electrical charge in the system.

The origin of charge lies in the atomic structure of matter. All matter is composed of atoms. The structure of an atom is represented in Figure 7. The nucleus contains most of the atomic mass in the form of neutrons and protons. The protons have a positive charge, while the neutrons are neutral (no charge). The nucleus is surrounded by a cloud composed of the much less massive electrons. Each electron has a negative charge of the same magnitude as the positive charge of a proton. A normal atom is neutral (has no net charge) because it has the same number of electrons and protons. Electrons are bound to the atom by electrical forces of attraction that obey the rule: unlike charges attract and like charges repel. These forces result in a voltage whenever the positive and negative charges are separated. Four common ways of creating a potential difference are listed in Table 3.

TABLE 3. METHODS USED TO PRODUCE A POTENTIAL DIFFERENCE	
Friction	Rubbing a cat's fur, for example, can create static electricity. Friction strips away electrons from the cat's fur, leaving a positive charge on the fur and a negative charge on the hand.
Chemical	Chemical energy in dry cells (flashlight battery type) or in wet cells (auto battery type) separates the charge to form a positive pole and a negative pole.
Magnetic Mechanical	A coil of wire moving in a magnetic field produces a force on electric charges that separates the positive and negative charges.
Light	Light energy falling on certain materials causes the separation of electrons from their atoms. Light meters make use of this process and indirectly record light intensities.

"Neutral" objects are those that contain the same quantity of positive and negative charge.

In Figure 8 some of the electrons have been moved from one object to another. The object with excess electrons is said to have a negative charge and the one from which the electrons have been removed, therefore, has a positive charge. If a path is provided for the electrons from one object to the other, an electric current will flow; that is, charge will be transferred from one object to the other until both have the same electrical potential. Potential difference moves charge in an electrical system in the same way that pressure moves fluid in a fluidal system. Thus, potential difference plays the role of a forcelike quantity in electrical systems.

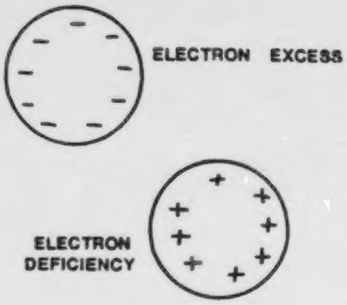
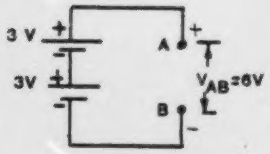


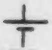
FIG. 8

Electric current is the rate at which charge moves through a system. Current exists in a circuit whenever two points of different electrical potential are connected by an electrical conductor. If there is no potential difference, the system is in equilibrium, and no charge flows.

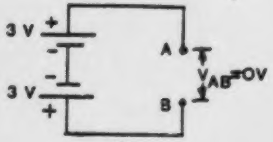
The potential difference between any two points in an electrical system is measured in volts (V). If a circuit contains more than one voltage source, the total voltage is the sum of the individual sources. In Figure 9a, two batteries are connected in series, with the positive terminal of one connected to the negative terminal of the other. In this circuit, both voltage sources will cause charge flow in the same direction if the circuit is completed. The total voltage produced is the sum of the individual voltages. In this case, a net potential difference acts as a forcelike quantity to cause charge flow.



(a)



SCHEMATIC SYMBOL FOR BATTERY



(b)

FIG. 9

In Figure 9b, the lower battery has been reversed. In this circuit, the two voltages oppose each other, and the total voltage is their difference, in this case zero. This zero value means that no charge will flow if points A and B are connected. The absence of a net potential difference means that no forcelike quantity is present to cause charge flow.

THERMAL SYSTEMS

The prime mover, or forcelike quantity, in thermal systems is temperature difference, often written as ΔT. Heat energy always flows naturally from warmer regions to colder regions; and the rate of heat flow in any system is dependent upon temperature difference, just as fluid flow is dependent upon pressure difference, and electric current is dependent upon voltage difference.

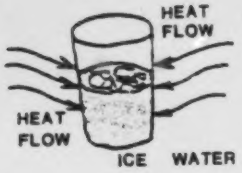


FIG. 10

A glass of water sitting in a warm room (Figure 10) will gain heat energy from its surroundings until all the ice melts and the water reaches the same temperature as the room. No further heat energy will flow in the final equilibrium condition, because no temperature difference exists.

All matter is composed of molecules (single atoms or groups of atoms) that are in constant motion. Heat energy is the energy of motion (kinetic energy) of moving molecules. This energy is transferred through a material by the action of the forces that exist between molecules of that material.

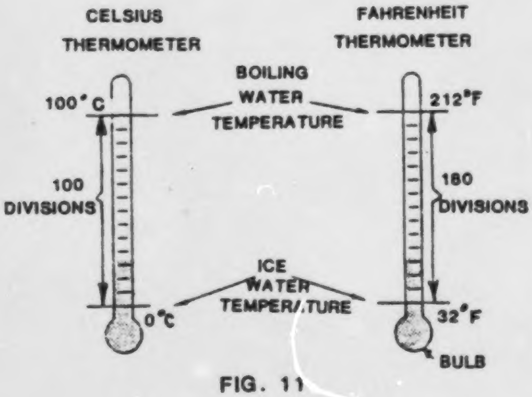


FIG. 11

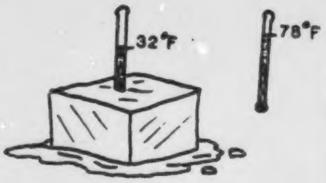
Temperature is a measure of the average kinetic energy of the molecules of a substance. The two most commonly-used temperature scales are shown in Figure 11. The Fahrenheit scale is a part of the English system of units; and the Celsius scale, sometimes called "centigrade," is a part of the SI system. Both systems use the freezing and boiling points of water as references, and divide the temperature difference between these points into an arbitrary number of divisions called degrees. Table 4 lists the formulas for conversion of temperature in one system to equivalent temperature in the other.

TABLE 4 . TEMPERATURE CONVERSION
Conversion of T°C to T°F:
$T^\circ F = 32^\circ F + 9/5\ T^\circ C$
Conversion of T°F to T°C:
$T^\circ C = 5/9\ (T^\circ F - 32^\circ F)$

When only the temperature difference is important, and not the actual temperature reading, conversion is made using the relationship 9F° = 5C°, or 1.8F° = 1C°. Temperatures are specified in degrees Celsius (°C) or degrees Fahrenheit (°F). Temperature difference is expressed in Celsius degrees (C°) or Fahrenheit degrees (F°), as shown in Example 1.

EXAMPLE 1 : TEMPERATURE CONVERSION

Given: A block of ice at 32°F is left in a room at 78°F.



Find:

(a) Temperature difference in Fahrenheit degrees.

(b) Temperature difference in Celsius degrees.

(c) Room temperature in degrees Celsius.

Solution:

(a) $\Delta T = T_2 - T_1$

$= 78°F - 32°F$

$\Delta T = 46F°.$

The temperature difference is 46 Fahrenheit degrees.

(b) $\Delta T\ (C°) = 5/9\ \Delta T\ F°$

$= 5/9\ (46F°)$

$\Delta T = 25.5C°$

The temperature difference is 25.5 Celsius degrees.

(c) $T\ (°C) = 5/9\ (78°F - 32°F)$

$= 5/9\ (46°F)$

$T = 25.5°C$

The temperature of the room is 25.5 degrees Celsius.

In thermal systems, temperature difference causes heat flow; therefore, temperature acts as a forcelike quantity.

SUMMARY

Forcelike quantities are the "prime movers" in all physical systems. Although one would be incorrect to say that net force, net torque, pressure differential, voltage, and temperature difference affect their respective energy systems in exactly the same way, they play strikingly analogous roles. The basic and very important similarities of these physical quantities can be of value in applying knowledge gained in one energy system to the solution of analogous problems in other energy systems. Table 5 summarizes the forcelike quantities in the four energy systems.

TABLE 5. FORCELIKE QUANTITIES

Energy System	Forcelike Quantity	Units		Quantity Moved
		English	SI	
MECHANICAL Translational	Force	lb	N	Mass moved through a distance
Rotational	Torque	ft•lb	N•m	Mass rotated through an angle
FLUIDAL	Pressure	lb/in^2	N/m^2	Fluid volume
ELECTRICAL	Potential Difference		V	Charge
THERMAL	Temperature Difference	F°	C°	Heat energy

MECHANICAL SYSTEMS, LINEAR MOTION

MATERIALS

Wood block (may substitute metal or plastic block) approximately 2" x 4" x 8" with cup hook.
Spring balance (pound or ounces and Newton calibration) with appropriate scale range.
Assorted weights (typical range) and weight hanger.
String.
Pulley and bracket for table mount.

PROCEDURE

This experiment demonstrates that forces in mechanics, when applied to objects, cause motion. Such occurrences are very common in our everyday lives, so nothing very startling is being demonstrated here. But, since the necessity arises to compare forcelike quantities in other energy systems, it is well to make a few simple observations concerning forces applied to masses. At the same time, the student will practice measuring forces and describing these measurements in appropriate units.

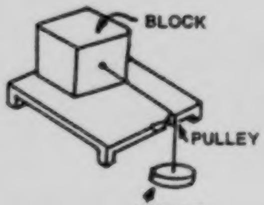


FIG. 12

1. Arrange the block, pulley and weight hangers as shown in Figure 12. Add gram weights to weight pan in small amounts until the block just begins to move (but not speed up) at a constant speed across table. A slight push may be necessary to start motion. Record weight in Data Table 1.

2. Arrange block and spring balance as shown in Figure 13. First, use a spring balance calibrated in pounds or ounces, then one in Newtons. Pull on spring balance carefully until block just begins to move at a constant speed. A slightly larger force may be required to start the motion than to maintain the motion at a constant speed. Record only the force necessary to produce a constant speed. Try this several times and record the average read-

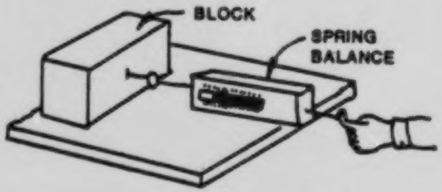


FIG. 13

ings (lb and Newtons) in Data Table 1.

3. Mathematically convert the force due to the gram weights (weights on weight hanger) to Newtons and the spring balance reading in pounds to Newtons. Record these values in Data Table 1. Calculate the percent difference between each of the forces in Newtons. Percent difference should be within limits specified by the instructor.

DATA TABLE 1. MECHANICAL LINEAR FORCES

Method Used	Measured Force	Converted Value In Newtons	% Difference
1. Block and weight on pan (gf)	____________gf	______________N	(1-2)______%
2. Block and spring balance (lbf)	____________lbf	______________N	(2-3)______%
3. Block and spring balance (Newtons)	____________N		(1-3)______%

MECHANICAL SYSTEM, ROTATIONAL MOTION

MATERIALS

Torque wrench (typical: 0 - 150 ft·lb).
Bolt and nut (typical: 3/8" or 1/2" diameter).
Socket for torque wrench and open-end wrench to fit bolt and nut.
Vice or other mounting device for bolt.

PROCEDURE

This experiment demonstrates the effect of applying a torque to a mechanical system. It shows that torque produces angular displacement, or rotational motion, and demonstrates that torque is the forcelike quantity in rotational motion.

1. Place a bolt in a vice or other suitable holder, and tighten a nut on the bolt to 20 ft•lb or more, using a torque wrench as illustrated in Figure 14. Loosen nut using torque wrench. Record torque wrench reading when nut is tightened and when it is loosened.

2. Again, tighten nut on bolt carefully to same torque reading as in number 1 above. Attach spring balance (which measures in pounds) to a wrench at a measured distance from center of the bolt, as in Figure 15. Record distance in feet. Pull on spring balance to loosen nut. Record reading on spring balance when nut is loosened.

3. Calculate torque value, and compare this value with value given on torque wrench before determining the percent difference between the two values. Percent difference should be within limits specified by the instructor. Record these values in Data Table 2.

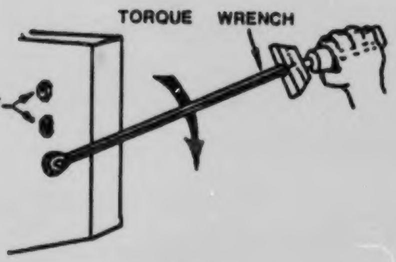


FIG. 14

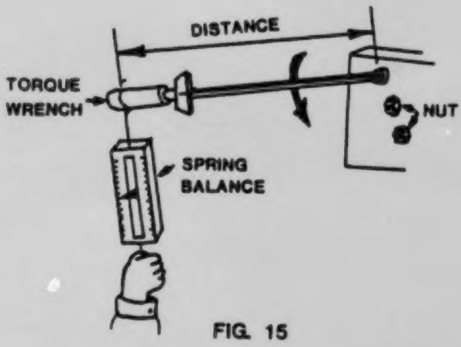


FIG. 15

DATA TABLE 2. MECHANICAL ROTATIONAL TORQUE

1. Torque to tighten nut ________ ft•lb
   Torque to loosen nut ________ ft•lb

2. Torque to tighten nut ________ ft•lb
   (reading on torque wrench)
   Length of handle ________ ft
   Spring balance reading ________ lb
   Calculated torque ________ ft•lb
   % Difference = ________ %

$$\% \text{ Difference} = \frac{\text{calculated-actual}}{\text{calculated}} \times 100$$

FLUIDAL SYSTEM

MATERIALS

Meter stick
U-tube manometer (approximately 50 cm high).

PROCEDURE

In the Discussion section of this module, pressure or pressure difference was introduced as the forcelike quantity in fluid systems. In the following experiment, the student will produce a pressure difference on a simple fluid system and will measure the pressure difference.

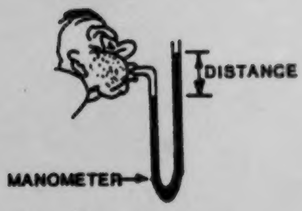


FIG. 16

1. Partially fill U-tube with water. Blow one end of tube (or use a squeeze bulb), and measure the maximum difference in levels of water (in inches), as illustrated in Figure 16.
2. The measured difference in levels is water pressure (in inches of water) that was produced by blowing. Convert value of pressure from inches of water to lb/in^2 (1 lb/in^2 is equal to 27.7 in of water). Record values in Data Table 3.

DATA TABLE 3. FLUID PRESSURE		
Pressure difference produced by blowing	________ in of water	________ lb/in^2

THERMAL SYSTEM

MATERIALS

Metal rod (typical 1/8" diameter x 12" copper).
Bunsen burner (may substitute electric hot plate).
Temperature stick (crayola can be used).
Stand or fixture to hold metal rod.

PROCEDURE

In thermal systems, a temperature difference serves as a forcelike quantity to cause heat energy to flow from regions of high temperature to regions of low temperature. In this experiment, a temperature difference is created in a metal rod when heat is applied to one end of the rod. The rod has been lined with wax material from a temperature stick. One then can "see" the heat flowing away from the hot end as the coating from the temperature stick changes its composition.

1. Draw a line with temperature stick along length of metal rod, and fasten rod in stand.

2. Bunsen burner at free end of rod, and adjust it so that the rod is about 3 cm above the Bunsen burner, as illustrated in Figure 17.

3. Light burner, and observe line from the heat stick melt progressively from hot zone to cold end. This melting illustrates the movement of heat down the length of the rod. At some point melting of the line may stop because heat in the rod is being drawn off by cold clamp and stand faster than it can be used to melt wax material on the outside of the rod.

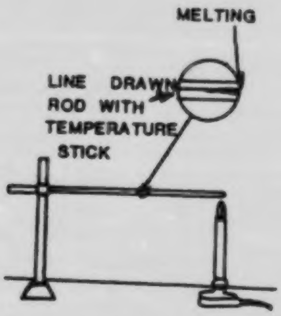


FIG. 17

4. Answer the following question:

   - How far from the Bunsen burner did the heat flow?_______.
   - In which direction does heat always flow? _________.
   - What forcelike quantity caused the flow of heat down the length of the rod? ________.

ELECTRICAL SYSTEM

MATERIALS

6. V d.c. source.
Capacitor (typical 1μF).
Voltmeter (0 - 10 V range, typical calibration).
Two alligator clip leads for voltmeter.

PROCEDURE

In the electrical system, voltage difference across battery terminals serves as the forcelike quantity, forcing charge to move when the positive and negative terminals are connected. Although movement of electrical charge is invisible, this experiment will show that a potential difference does move charge when connected in a circuit.

1. Use voltmeter to measure emf of battery. Record reading in Data Table 4. Also check capacitor with voltmeter, and record this reading. The capacitor reading should be zero. Next, connect leads from battery to capacitor; wait a few seconds, and disconnect leads. (What has the connection of the battery to the capacitor accomplished?)

2. Attach leads from voltmeter to capacitor, and record initial voltage reading. Watch voltage decay. This represents movement of charge from negative side of capacitor to the positive side. Record these readings in Data Table 4.

DATA TABLE 4. POTENTIAL DIFFERENCE	
Potential Difference Before Charging Capacitor	Potential Difference After Charging Capacitor
Battery emf ________volts	Initial reading ______volts
Capacitor emf _______volts	Final reading _______volts
What showed that charge moved in the capacitor during the decrease in voltage? ____________	

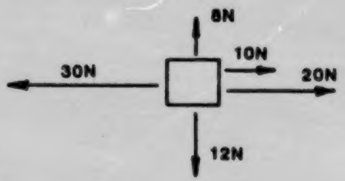
SELF CHECK

1. Define the following physical quantities; and, where applicable, state their units in both the SI and English systems.

   a. Force.
   b. Torque.
   c. Pressure.
   d. Voltage.
   e. Temperature difference.

2. An object is acted upon by three simultaneous forces: 40 lb to the right, 180 lb to the left, and 200 lb to the right. Determine the resultant force and the direction of motion.

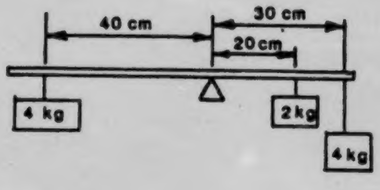
3. The diagram at the right shows five forces acting upon an object initially at rest. Determine the net force and the direction of motion.



4. A force of 10 lb is used to produce a torque of 32.8 ft·lb. Determine the lever arm.

5. Convert the values in problem 4 to metric units, and solve for lever arm in meters. Compare answer to that from problem 4.

6. Three masses are hung from a meter stick, as shown at the right. Find the net torque and the direction of rotation.



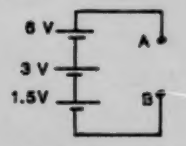
7. A 130 lb lady wearing spike heel shoes, with heels measuring 1/2" x 1/2", is standing on a linoleum floor. Calculate how much pressure will be applied to the linoleum if she puts all her weight on one heel.

8. Calculate the water pressure in pounds per square inch at the bottom of a lake 250 ft deep. (Weight density of water is 62.4 lb/ft^3.)

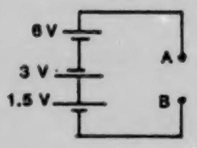
9. Calculate the depth of water in a tank if the pressure at the bottom is 10^5 N/m^2. (Density of water is 1000 kg/m^3.)

10. Determine the total voltage V_{AB} in each of the following circuits:

(A)



(B)



11. Convert each of the following temperatures in °F to temperatures in °C.

a. 413°F.
b. 26°F.
c. -13°F.

12. Convert the following temperatures in °C to temperatures in °F.

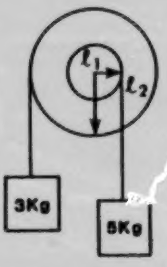
a. 25.5°C.

b. 5°C.
c. -40°C.

13. Describe in a short paragraph the similarities of forcelike quantities in the four energy systems discussed in this module.

14. Describe the conditions necessary for equilibrium in each energy system.

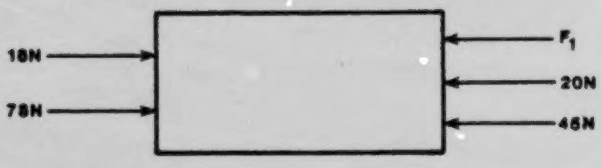
15. A frictionless pulley has a 3 Kg and a 5 Kg mass attached as shown below. Calculate the resultant torque and determine the direction of rotation.



ℓ_1 = 0.25 m
ℓ_2 = 0.5 m

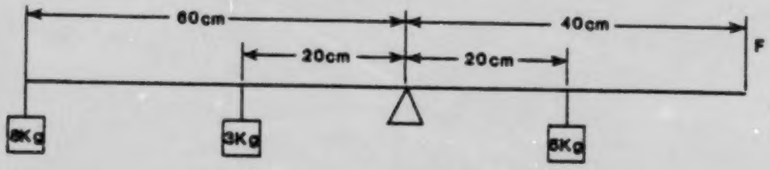
16. What is the average pressure on the face of a dam if the water exerts a total force of 12×10^8 Newtons and the surface area of the dam is 600 square meters?

17. The diagram below shows five forces acting on the same object. Calculate what the unknown force (F_1) must be if the object is to remain at rest if it was originally at rest before the forces were applied.



18. What is the lifting capacity of a hydraulic jack if the fluid pressure at maximum load is 1600 lb/in^2 and the piston which lifts the load has a radius of 2 inches?

19. Three masses are hung from a massless meter stick as shown below. What force must be applied to the right end of this stick in order to keep the meter stick from rotating? Be sure to specify the direction in which the force must be applied.

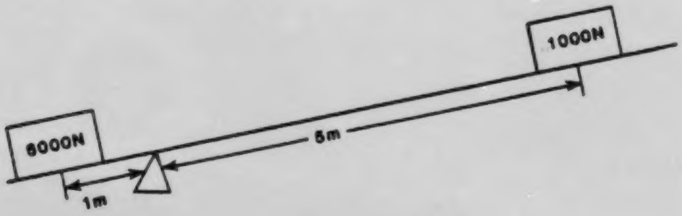


20. How much would a man have to weigh in order to, when standing on one foot, exert the same pressure on the floor as did the lady in Problem 7. Use 18 in^2 as the area of the bottom of the man's foot. Considering the necessary weight of the man, is it reasonable to assume that a man could exert the same pressure as can the lady?

21. What is the density of a liquid if, at the bottom of a tank filled with this liquid, the pressure is 10^5 N/m^2. The depth of the tank is 6 meters.

22. The porthole of a small submarine breaks when an average force of 10^6 N is distributed over its area. How deep below the surface of the ocean can this submarine go before the glass in the porthole breaks? The area of the porthole is 2 square meters.

23. A lever is to be used to lift an object weighing 6000 N. If the distance from the pivot to the object is 1 meter and the lever is 6 meters long, can the object be lifted if the maximum force which can be applied to the other side of the lever is 1000 N?



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See especially Chapters 4, 9, 13 and 20.

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WORK

CHAPTER 2-0

Years ago, most work accomplished was through physical efforts of men and animals. Today, less than three percent of the energy required by industry is provided by muscular exertion. Modern industries are highly mechanized. Even many machines are controlled by mechanical devices rather than by men. "Work" has become a technical term, and scientists and technicians must define it in a technical way. In this module, work will be defined, and various applications of work in the fields of mechanical, fluidal, thermal and electrical energy will be shown. Energy will be defined also, and units for work and energy will be introduced.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define work and energy in general terms that apply to any energy system, and distinguish work from energy in the following systems:

  Mechanical translational.
  Mechanical rotational.
  Fluidal.
  Electrical.
  Thermal.

- Define the following units of work and energy:

  Foot-pound.
  Newton-meter.
  Joule.
  Calorie.
  British Thermal Unit.

- Define the following terms and explain their usefullness in determining work done:

  Radian (mechanical system).
  Current (electrical system).
  Specific heat (thermal system).
  Heat capacity (thermal system).

- Given two of the following quantities in a mechanical translational system, determine the third:

  Force.
  Displacement.
  Work.

- Given two of the following quantities in a mechanical rotational system, determine the third:

  Torque.
  Angular displacement.
  Work.

- Given two of the following quantities in a fluidal system, determine the third:

  Pressure difference.
  Volume displaced.
  Work.

- Given two of the following quantities in an electrical system, determine the third:

  Voltage.
  Charge transferred.
  Work.

- Given the temperature difference across a uniform thickness of a substance, the dimensions of the substance, and its thermal conductivity, calculate the heat flow rate through the substance.

- Given two of the following quantities in a thermal system, determine the third:

  Temperature change of object.
  Heat capacity of object.
  Work (heat energy transferred).

- Define and give examples of:

  Latent heat.
  Sensible heat.

- State the general equation for work, and explain how it applies to each of the following energy systems:

  Mechanical translational.
  Mechanical rotational.
  Fluidal.
  Electrical.
  Thermal.

WORK AND ENERGY

In Module 1-0, "Force," forcelike quantities were defined as those physical quantities that produce motion in mechanical, fluidal, electrical and thermal systems.

If no net force, net torque, pressure difference, voltage difference or temperature difference is present, the system is in equilibrium and there will be no change in the system.

The forcelike quantities in a system in equilibrium are balanced in such a way that they produce no change. In mechanical systems, this state of balance occurs when the sum of all forces and torques is zero, which means that objects at rest remain at rest and that objects in motion continue the same motion with no change. In fluidal systems, equilibrium occurs when the pressure difference in the fluid is zero, indicating that there is no fluid flow. In electrical systems, no charge moves between two points if the sum of the voltages between them is zero. Thermal equilibrium exists in a closed system only when there is no temperature difference in that system. This condition means that heat energy is not moving in the system. A system in equilibrium experiences no change, although separate forcelike quantities (which add to zero collectively) are present in the system.

If equilibrium does not exist in a system, that system changes in response to the unbalanced forcelike quantities present. Work is done any time that a forcelike quantity produces a change in a system. The amount of work done depends upon both the magnitude of the forcelike quantity that produces the change and the magnitude of the change produced. Work is a description of change in a system, and occurs only when the system is not in equilibrium.

Energy is the ability to do work. Energy is present whenever forcelike quantities are present, even if the system is in equilibrium. When any physical system in equilibrium is changed in such a way that it no longer is in equilibrium, energy is converted to work, and the system changes in some way until a new equilibrium condition is reached. Work and energy are measured in the same units, and the total amount of "work done" always equals the total amount of "energy used." This principle is called "the law of conservation of energy." It means that energy is neither created nor destroyed, but is only changed from one form to another by the process of doing work — or causing change — within a system.

The general formula for determining the amount of work done in any system is given in Equation 1.

$$\boxed{\text{WORK} = \text{FORCELIKE QUANTITY} \times \text{DISPLACEMENTLIKE QUANTITY}} \qquad (1)$$

In the remainder of this module, this equation is applied to mechanical, fluidal and electrical systems. The discussion of thermal systems will show that this general equation is not useful in describing heat energy.

MECHANICAL SYSTEMS

MECHANICAL WORK, TRANSLATIONAL

In the mechanical translational system, the general equation for work takes the form of Equation 2.

$$W = F\ell \qquad (2)$$

where

W = mechanical work done by a force.
F = magnitude of component of force acting in the direction of motion.
ℓ = distance through which force acts.

This equation defines only the work done by a force. It does not indicate the final form taken by the energy change produced, but only that a mechanical force has done work to change the form of energy in a system. According to this definition of work, the object must be moved, or no work is done.

In Figure 1, a man has done mechanical work by lifting a weight from the floor. The force exerted by his muscles has moved the weight through a distance. If he had not been able to lift the weight, no work would have been done. The man can tug and strain at the weight; but if he is unable to lift it, he does no work on the weight. When he holds the weight above his head, he is doing no work on the weight, because it is not being moved by a force. How tired he becomes is not involved in the definition of mechanical work. The energy expended by his muscles in holding the weight is converted to heat and to chemical energy, not to mechanical work.

MAN LIFTING A 200 lb WEIGHT

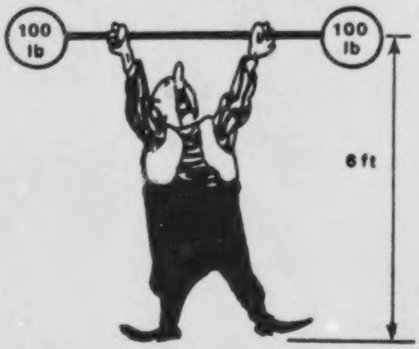


FIG. 1

Example A illustrates the use of Equation 2 in determining work.

EXAMPLE A: WORK DONE LIFTING A WEIGHT.

Given: A man lifts a 200-lb weight 6 ft.

Find: Mechanical work done.

Solution:

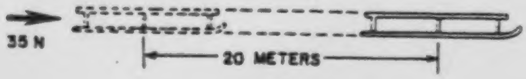
$$W = F\ell$$
$$= (200\ \text{lb})(6\ \text{ft})$$
$$W = 1200\ \text{ft}\cdot\text{lb}$$

The unit of mechanical work in the English system is the foot-pound — the amount of work done when an unbalanced force of one pound is applied through a distance of one foot. Notice that the unit of work (ft•lb) is the product of the units of force and distance.

In the SI system, the same relationship holds, and the unit of work is the Newton-meter (N•m). This unit is equivalent to one Joule (J) of energy required for accomplishing the work, as shown in Example B.

EXAMPLE B: WORK DONE PUSHING A SLED

Given: A horizontal force of 35N is applied continuously to push a sled through a distance of 20 m.



Find:
(a) Work done
(b) Energy used

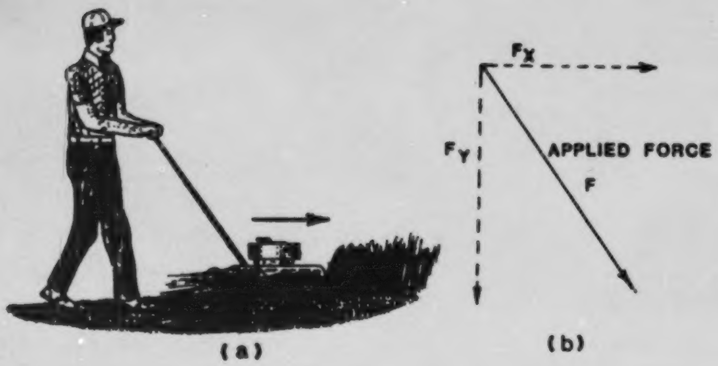
Solution:

(a)
$$W = F\ell$$
$$= (35\ \text{N})(20\ \text{m})$$
$$W = 700\ \text{N}\cdot\text{m}$$

700 N•m of work is done in moving the sled.

(b) Since 1 N•m = 1 J, 700 J of energy was expended in moving the sled.

In determining the work done by a force, only the component of the force in the direction of motion is considered. In Figure 2a, a man pushes a lawnmower. The direction of the force applied to the mower handle and the direction of motion of the mower are not the same. Figure 2b is a vector diagram showing the horizontal (F_x) and vertical (F_y) components of applied force (F). The vertical component of the force does no work, because there is no vertical motion of the mower. Only the horizontal component does work, and its magnitude is used in Equation 2 for determining work. Only the component of the resultant force that lies in the same direction as the resulting motion does work.



(a) (b)

FIG. 2

MECHANICAL WORK, ROTATIONAL

In the mechanical rotational system, the general equation for work takes the form of Equation 3.

$$W = \tau\theta \qquad (3)$$

where

W = mechanical work done by a torque.
τ = magnitude of torque producing work.
θ = angle of rotation through which torque is applied.

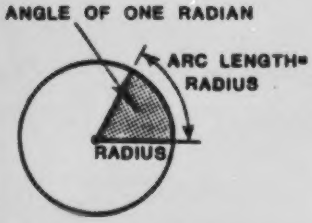


FIG. 3

Angles often are measured in degrees or in number of revolutions; however, a more convenient measure in radians also is commonly used. In Equation 3, the angle must be expressed in radians. An angle of one radian is that angle which, if placed at the center of a circle, would intercept an arc length equal to the radius of the circle. The size of an angle of one radian is shown in Figure 3. This angle of one radian is approximately 57.3°, but an identity derived from the formula for the circumference of a circle is easier to remember. The circumference of a circle encloses an angle of 360°. Since circumference is found by multiplying 2π times radius, and an arc length of one radius corresponds to an angle of one radian: 2π radians = 360°. This relationship and another that results when both sides are divided by two are shown in Table 1.

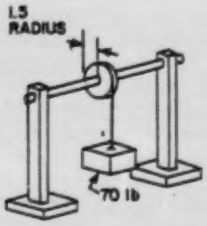
TABLE 1: CONVERSION FACTORS

$$2\pi \text{ radians} = 360^\circ$$

$$\pi \text{ radians} = 180^\circ$$

The radian is not actually a unit in the same sense as a meter or a pound. It is the ratio of the length of an arc to the length of the radius. Lengths are expressed in the same units, which cancel to leave a dimensionless quantity. Thus, when angles expressed in radians are used in calculations, the term "radian" is dropped, as illustrated in Examples C and D.

EXAMPLE C: WORK DONE TO TURN A PULLEY



Given: A force of 70 lb is required to turn a pulley with a radius of 1.5 feet at a constant speed. The pulley turns through 5 revolutions.

Find: Work done on pulley.

Solution: Recall from Concept Module 1-0.

$$\tau = F\ell \text{ (Torque = Force x lever arm)}$$

$$= (70 \text{ lb})(1.5 \text{ ft})$$

$$\tau = 105 \text{ ft}\cdot\text{lb}$$

$$\theta = (5 \text{ rev})\left(\frac{2\ \pi \text{ rad}}{1 \text{ rev}}\right) = 10\pi \text{ rad}$$

From Equation 3:

$$W = \tau\theta$$

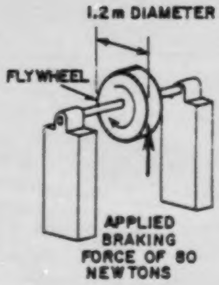
$$= (105 \text{ ft}\cdot\text{lb})(10\pi) \text{ ("rad" is dropped.)}$$

$$= (105 \text{ ft}\cdot\text{lb})(10)(3.14)$$

$$W = 3297 \text{ ft}\cdot\text{lb}$$

3297 ft·lb of work is done by the applied torque.

EXAMPLE D: ENERGY STORED IN A ROTATING FLYWHEEL



Given: A heavy flywheel, 1.2 meters in diameter, rotates 484 degrees before stopping, while a braking force of 80 Newtons is being applied at its outer edge.

Find: Amount of energy consumed in stopping rotation.

Solution: Since diameter = 1.2 meters, radius = 0.6 meters. Energy consumed = work done.

$$W = \tau\theta = F\ell\theta$$

$$= (80\ \text{N})(0.6\ \text{m})(484^\circ)\left(\frac{\pi\ \text{rad}}{180^\circ}\right)$$

$$= (48\ \text{N}\cdot\text{m})(8.45\ \text{rad})$$

$$W = 405.5\ \text{N}\cdot\text{m}$$

Energy required to stop flywheel is 405.5 N·m or 405.5 J.

Notice that the units of torque and work are the same — N·m in SI units and ft·lb in English units. This does not indicate that torque and work are equivalent. Care must be taken to identify a quantity expressed in these units as either torque or work. The multiplication of torque (N·m or ft·lb) by an angle of rotation in radians (no units) yields work (N·m or ft·lb). In the SI system, both work and energy usually are expressed in Joules to eliminate the possible misunderstanding.

FLUIDAL SYSTEMS

In a fluidal system, work is accomplished when a pressure is applied to move a volume of fluid. The general formula — Work = Forcelike Quantity x Displacementlike Quantity — takes the form shown by Equation 4 for the fluidal system.

$$W = (\Delta p)V \qquad (4)$$

where

W = work done moving fluid.
Δp = pressure difference through which fluid moves.
V = volume of fluid transferred.

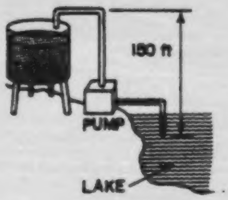
In the fluid system, the forcelike quantity (quantity that causes movement of the fluid) is pressure, and the displacementlike quantity is represented by the volume of fluid moved.

In a system involving liquids, which are incompressible, the volume of a given mass of fluid remains the same; that is, the volume of fluid entering a pipe is equal to the volume leaving the pipe. Volume of fluid is constant, and only fluid pressure changes in the system. Example E illustrates the use of Equation 4 in solving a problem.

EXAMPLE E: WORK DONE TO FILL A WATER TANK

Given: Water tank system of 500 cubic feet. The density of water is 62.4 lb/ft^3.

Find: Amount of energy required to fill it with water from the lake 150 ft below.



Solution: From Concept Module 1-0

Pressure = height x density

$$p = h\rho_w$$

$$= (150\ ft)(62.4\ lb/ft^3)$$

$$p = 9360\ lb/ft^2$$

The pressure difference Δp between the top of the tank and the surface of the lake is 9360 lb/ft^2.

$$W = (\Delta p)V$$

$$= (9360\ lb/ft^2)(500\ ft^3)$$

$$W = 4{,}680{,}000\ ft \cdot lb$$

The work required to fill the tank is 4.68×10^6 ft·lb.

In cases where the fluid is a compressible gas, the volume change of the gas must also be taken into account when work is calculated.

As previously stated, there must be some movement in the system, or work has not been done. For example, air pressure of forty pounds per square inch in a tire is not work in itself, but work is done in putting air into the tire. The change in tire volume involved in the work is the difference between the volume of air under pressure in the tire and the volume that the same quantity of air would occupy if released to the atmosphere. The pressure involved is the pressure of the air in the tire at any time, which increases gradually from one atmosphere ($14.7\ \mathrm{lbs/in^2}$) to the final pressure in the tire.

Work done in filling a tire with air is stored in increased thermal energy of the air molecules within the tire as indicated by a rise in temperature and in the stretched tire itself. A mechanic must exercise caution in filling an automobile tire (or tube) with air after repairing it, because this stored energy can be released quickly. If the tire and wheel are not put together correctly, the tire can explode and cause serious injury. Businesses involved in changing and repairing large tires provide a heavy safety frame that holds the tire while it is being filled with air.

ELECTRICAL SYSTEMS

In an electrical system, work and energy are measured in Joules (J). The general formula for work takes the form shown in Equation 5.

$$W = VQ \tag{5}$$

where
V = potential difference through which charge is moved.
Q = quantity of charge moved.

Potential difference, measured in volts (V), is the forcelike quantity. Charge, measured in coulombs (C), is that substance in electrical circuits that is moved by a potential difference. Thus, charge is the displacementlike quantity.

A net charge of one coulomb represents 6,250,000,000,000,000,000 more electrons at the negative pole than at the positive pole! Since this number is so large, it usually is written with the aid of scientific notation as 6.25×10^{18}. One Joule of work or energy in the electrical system is the amount of energy required to move one coulomb of charge through a potential difference of one volt. Table 2 gives useful conversion factors for electrical energy.

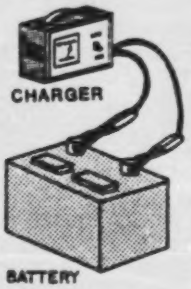
TABLE 2: CONVERSION FACTORS FOR ELECTRICAL SYSTEMS

1 Joule = 1 volt × 1 coulomb

1 coulomb = charge of 6.25×10^{18} electrons

Example F shows the use of Equation 5 in solving a problem.

EXAMPLE F: ENERGY REQUIRED TO CHARGE A BATTERY



Given: A 6-volt automobile battery stores 8000 coulombs.

Find: Energy required to charge it.

Solution:

$$W = VQ$$

$$= (6\ \text{V})(8000\ \text{C})$$

$$W = 48{,}000\ \text{J}$$

48,000 J or 48 kJ are required to charge the battery.

In most electrical systems, the measurement of electrical charge is inconvenient. Instead, electrical current is measured. Current is the rate at which charge flows, and is defined by Equation 6.

$$I = \frac{Q}{t} \tag{6}$$

where

I = current in amperes (A).

Q = charge transferred in coulombs (C).

t = time in seconds (s) required for transfer.

This equation will be examined in more detail in Concept Module 3-0, "Rate." It is presented here only because of its importance in determining work done in an electrical system. Example G shows the use of Equations 5 and 6 in solving for electrical work.

EXAMPLE G: WORK DONE BY AN ELECTRIC MOTOR

Given: A d.c. electric motor, operating on a voltage of 12 V and drawing a current of 4A, runs for 2 minutes.

Find: Electrical energy consumed.

Solution: From Equation 6:

$$Q = It$$
$$= (4\ \text{A})(120\ \text{s})$$
$$Q = 480\ \text{C}$$

The charge transferred is 480 coulombs.

From Equation 5:

$$W = VQ$$
$$= (12\ \text{V})(480\ \text{C})$$
$$W = 5760\ \text{J}$$

THERMAL SYSTEMS

In a thermal system, application of the general work equation — Work equals Forcelike Quantity times Displacementlike Quantity — does not follow the analogy described for the other systems. In thermal systems, the forcelike quantity is identified as a temperature change, and the displacementlike quantity as heat energy. To this point, these analogies provide a systematic unification with the other energy systems. In the application of the work equation, however, the analogy fails. Like all analogies, there may be a situation in which the point of view of the analogy is incapable of clearly describing the situation. Yet in thermal systems, it is quite evident that a temperature difference will cause heat energy to flow and thus, even here, it is useful to identify a forcelike quantity (ΔT) and a displacementlike quantity (ΔH). The analogy is helpful, even though the general work equation (Work = Forcelike Quantity x Displacementlike Quantity) does not hold. Consider, for example, heat flow through a slab of glass (windowpane) or sheet of corkboard as shown in Figure 4. The equation which governs the rate of heat flow through a given thickness of substance, due to a temperature difference ΔT across opposite faces of the substance, is given by Equation 7.

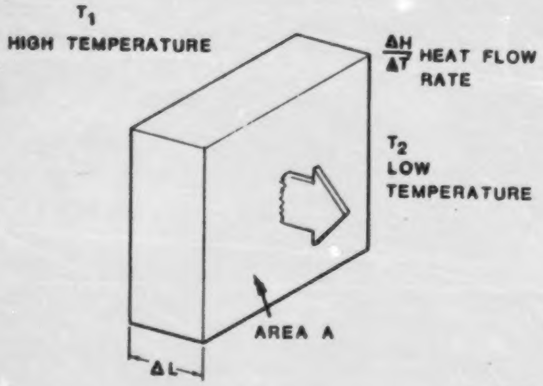


FIG 4

$$Q_H = KA\frac{\Delta T}{\Delta L} \qquad (7)$$

where

ΔT = temperature difference across the substance.

ΔL = uniform thickness of substance.

A = area of substance.

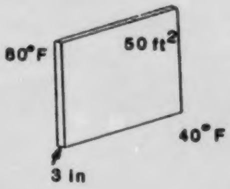
K = thermal conductivity (a constant characteristic of the substance).

Q_H = the heat flow rate (Btu/hr) caused by the temperature difference ΔT across the substance.

It is in thermal systems such as that described by Equation 7 that one sees a compelling similarity between temperature difference ΔT in thermal systems and pressure difference ΔP in fluidal systems or voltage difference ΔV in electrical systems. Each forcelike quantity moves something. In fluidal systems, a mass or volume of fluid is moved. In electrical systems, a quantity of charge is moved. And in thermal systems, a quantity of heat energy is moved. Example H illustrates the use of Equation 7 in a practical problem.

EXAMPLE H: HEAT FLOW THROUGH CORKBOARD

Given: A sheet of corkboard 3 inches thick and 50 ft^2 area. One side of corkboard is at 80°F, the other at 40°F. Thermal conductivity of corkboard is $0.30\ \frac{\text{BTU}\cdot\text{in}}{\text{hr}\cdot\text{ft}^2\cdot\text{F}^\circ}$.



Find:

(a) Forcelike quantity ΔT

(b) Rate of heat energy flow Q_H

Solution:

(a) $\Delta T = 80^\circ\text{F} - 40^\circ\text{F} = 40\ \text{F}^\circ$ (forcelike quantity)

(b) $Q_H = KA\frac{\Delta T}{\Delta L}$ (From Equation 7)

$$= \left(\frac{0.30\ \text{BTU}\cdot\text{in}}{\text{hr}\cdot\text{ft}^2\cdot\text{F}^\circ}\right)\left(50\ \text{ft}^2\right)\left(\frac{40\ \text{F}^\circ}{3\ \text{in}}\right)$$

$$Q_H = 200\ \frac{\text{BTU}}{\text{hr}}$$

Equation 7 and Example H are concerned with thermal systems where a constant temperature difference is maintained and, as a consequence, heat energy flows from the hot side of the body to the cold side. Here the constant temperature difference ΔT can be viewed as causing heat to flow. In other thermal systems, such as that illustrated by heating a pot of water on a stove, the temperature of the water changes as heat energy moves from the hot stove into the water. In this instance, it is the temperature difference ΔT between the hot stove and cool water that moves heat energy into the water, and as a result, the cool water undergoes a temperature change of its own. The temperature change of the water can be determined from knowledge of the heat energy flowing into the water and properties of the water alone, without knowledge of the temperature difference ΔT between stove and pot.

For example, the quantity of heat energy required to change the temperature of a body (such as the pot of water) is given by Equation 8.

$$H = mc\,\Delta T \qquad (8)$$

where

H = thermal energy added to the body to cause a temperature change ΔT.
ΔT = temperature change of the body.
m = mass of body that undergoes change ΔT.
c = constant characteristic of the body known as the specific heat of the substance.

In this equation, the quantity mc is the product of the mass and the specific heat of the body that undergoes the temperature change. The product mc is called the "heat capacity" of the object and represents the amount of heat energy required to raise the temperature of that body by one degree (Fahrenheit in English units and Celsius in SI units).

In thermal systems, the quantity of heat energy that flows from higher to lower temperature regions is driven by the temperature difference. In essence, heat energy transferred to a substance actually is the increased motion of the molecules in the substance. Thermal properties of different substances vary widely and depend intimately upon molecular structure of the substances.

Specific heat indicates how many units of heat energy are required to raise a unit mass of some substance a unit temperature difference. Water, for example, has a specific heat of 1 cal/g·C°, or 1 Btu/lb·F°. Since water has a specific heat — that is, an ability to soak up large amounts of heat energy at the expense of only a small increase in temperature — it makes an ideal coolant.

That's why water is used as the coolant in most systems in which considerable thermal energy is generated.

In SI units, quantity of heat is measured in calories (cal) or kilocalories (kcal). One calorie is the amount of heat required to raise one gram of water one degree Celsius. Since the calorie measures small amounts of heat, kilocalories (kcal) often are used to measure larger quantities. Measures of the heat values of foods, incorrectly given as calories in diet books, actually are kilocalories. For instance, bread claimed to have 60 calories per slice actually has 60 kilocalories per slice.

In English units, quantity of heat is measured in British thermal units (Btu). One British thermal unit is the amount of heat required to raise one pound of water one degree Fahrenheit. This unit is used in the United States to measure cooling capacity of air conditioners or heating capacity of stoves. Table 3 lists some useful conversion factors.

TABLE 3: CONVERSION FACTORS

1 Btu = 252 cal
1 kcal = 1000 cal
1 cal = 3.09 ft•lb
1 Btu = 778 ft•lb
1 cal = 4.18 Joules

Specific heats of numerous substances have been determined experimentally and can be found in tables such as Table 4.

TABLE 4: SPECIFIC HEATS OF COMMON SUBSTANCES

Substance	Specific Heat
Air	0.24
Aluminum	0.22
Brass	0.091
Copper	0.093
Glass	0.21
Iron (steel)	0.115
Stone (average)	0.192
Tin	0.055
Water	1.00
Wood (average)	0.42

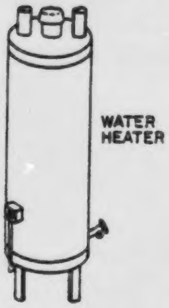
Specific heats are stated in Table 4 as the ratio of the specific heat of the given substance to that of water. The value of specific heat of any substance is the same in either SI or English units. Units for specific heat in Table 4 are Btu/lb·F°, kcal/kg·C° or cal/g·C°. Specific heat of water was adopted as a standard because of its large heat capacity and availability.

Use of Equation 8 in thermal systems is illustrated in Examples I and J.

EXAMPLE I: ENERGY REQUIRED TO HEAT WATER

Given: 417 pounds (approximately 50 gallons) of water is heated from 70°F to 130°F ($1 \text{ ft}^3 = 7.48 \text{ gal}$).

Find. Amount of heat energy required. Assume no heat loss in heating water tank or atmosphere.



Solution: Equation 7

$$H = mc\Delta T$$

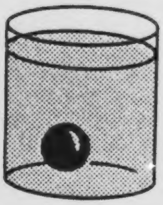
$$= (417 \text{ lb})\left(1 \frac{\text{Btu}}{\text{lb}\cdot\text{F}^\circ}\right)(130^\circ\text{F} - 70^\circ\text{F})$$

$$= (417 \text{ lb})\left(1 \frac{\text{Btu}}{\text{lb}\cdot\text{F}^\circ}\right)(60\text{F}^\circ)$$

$$H = 25{,}020 \text{ Btu}$$

EXAMPLE J: HEAT ENERGY REMOVED TO COOL AN OBJECT

Given: A 25-kg brass ball at 100°C has cooled to 30°C in an insulated container of water. The water temperature increases from 10°C to 30°C.



Find:
(a) Amount of heat energy removed from the ball and added to the water.
(b) Mass of water in container.

Assume no heat loss to container or atmosphere.

Solution: (a) $H = mc\Delta T$

$$= (25\ \text{kg})(0.091\ \frac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ})(100^\circ\text{C} - 30\ \text{C}^\circ)$$

$$H = 159.3\ \text{kcal, or } 159{,}300\ \text{cal}$$

(b) $m = \frac{Q}{c\Delta T}$

$$= \frac{(159{,}300\ \text{cal})}{(1\ \frac{\text{cal}}{\text{g}\cdot\text{C}^\circ})(20\text{C}^\circ)}$$

$$m = 7965\ \text{g}$$

CHANGE OF STATE

If heat is supplied to matter, two possible changes can occur. The first possible change, described above, is that the temperature of the material will be raised while its state remains fixed. The second possible change is that the temperature will remain fixed, but the state of the substance will change; that is, the substance will change from a solid to a liquid, or from a liquid to a gas. Heat can change the temperature or the state, but not both at the same time.

If the temperature of a material is changing, heat supplied to the material is classified as "sensible heat," since the effect can be sensed by means of a thermometer. If the state of the substance is changing, the heat is termed latent heat. The word "latent" means "hidden." Figure 5 shows a container of ice, to

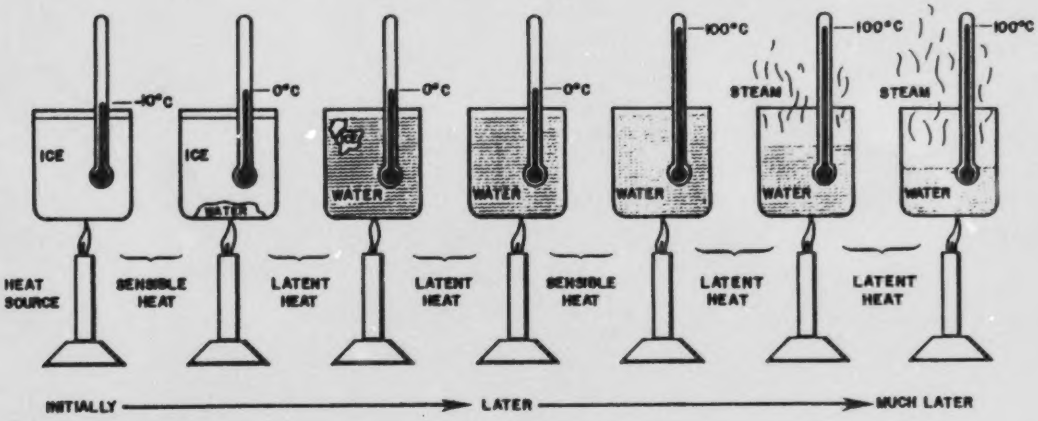

-10°C
0°C
0°C
0°C
100°C
100°C
100°C
ICE
ICE
ICE
WATER
WATER
WATER
WATER
WATER
WATER
STEAM
STEAM
HEAT SOURCE
SENSIBLE HEAT
LATENT HEAT
LATENT HEAT
SENSIBLE HEAT
LATENT HEAT
LATENT HEAT
INITIALLY
LATER
MUCH LATER

FIG. 5

which heat is applied. The resulting temperatures at various times in the heating process are shown. Latent heat is energy used in changing matter from one state to another. Change of state is a very important principle used in air conditioners, in refrigerators, and in many other cooling and heating systems. Calculations involving latent heat are included in application modules.

SUMMARY

Work is done in any physical system whenever a forcelike quantity causes a change (displacementlike quantity) within the system. Energy is the ability to do work, and is conserved within a closed system. When a system is in equilibrium, it contains energy, but no work is being done, because no displacement is occurring as the result of forcelike quantities within the system. When a system is not in equilibrium, displacement of some kind occurs as the result of forcelike quantities; and energy is converted to work within the system. Work and energy are equivalent and are measured in the same units. Table 5 shows the general work equation and the form it takes in mechanical, fluidal and electrical systems. The equation for heat energy required for a temperature increase also is included, although it is not derived from the general work equation.

TABLE 5: WORK AS A UNIFYING CONCEPT

ENERGY SYSTEM	WORK = FORCELIKE QUANTITY x DISPLACEMENTLIKE QUANTITY
MECHANICAL	
Translational	Work = Force x Displacement (in direction of force)
Rotational	Work = Torque x Angular displacement
FLUIDAL	Work = Pressure Difference x Volume change
ELECTRICAL	Work = Potential Difference x Charge transferred
THERMAL	Heat Energy = Temperature change x Heat capacity

MECHANICAL SYSTEM. LINEAR WORK

MATERIALS

Mass (can be cardboard box with something heavy in it) approximately 10 kg.
Balance.
Meter stick.

PROCEDURE

In this experiment, the work done in lifting an unknown mass and placing it on a table is calculated.

1. Lift unknown mass from floor onto scale (Figure 6).
2. Determine mass of object with a balance scale.
3. Measure distance object was lifted to place it on table.
4. Fill in Data Table 1; calculate work done in lifting mass to table.

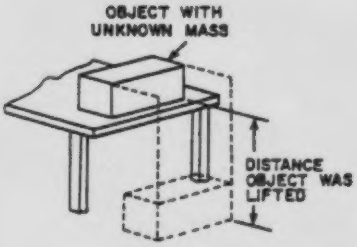


FIG. 6

DATA TABLE 1

Mass of Object Lifted	Distance Object Was Lifted	Work Done Lifting Mass
		N-m
		J
		ft•lb

MECHANICAL SYSTEM, ROTATIONAL WORK

MATERIALS

Gyroscope (lab model or toy).
Pulley and bracket for table mount.
Micrometer.
200 g mass.
String.

PROCEDURE

Determine work done in spinning gyroscope illustrated in Figure 7.

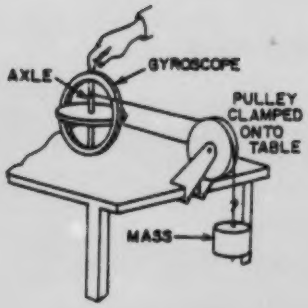


FIG.7

1. With a micrometer, measure diameter of gyroscope axle. Calculate radius of axle.

2. Count number of revolutions used in winding up string. Do not tie string to axle.

3. Hold gyroscope carefully, and place string across a pulley mounted on table. Suspend a mass of approximately 200 g on string so that when string is released, weight will fall toward floor, in such a manner that a constant force is applied on string. String should be completely unwound before weight strikes floor.

4. Fill in Data Table 2, calculate torque on axle and work done in spinning gryoscope wheel.

DATA TABLE 2

Axle		Number of		Mass on String	Force on String (N)	Torque	Work
Diameter	Radius	Revolutions	Radians				
____ mm ____ m	____ m			____ g	____N	____N-m	____N-m

Recall that 2 π rad = 1 revolution.

Recall that 1 kg weighs 9.8 N.

FLUID SYSTEM, WORK

MATERIALS

Rubber hose.
2 hose clamps.
Pressure gage (0-100 psi).
Gallon jug (such as a milk jug).
Beaker, graduated in liters.

PROCEDURE

In this experiment, work done in filling a gallon container from a water faucet will be determined.

1. Insert one end of a rubber hose on a pressure gage, and clamp securely. Clamp other end of hose securely to water faucet. Turn on water fully, and measure water pressure at faucet.

2. Fill jug with water from faucet.

3. Measure water by pouring it into a beaker graduated in liters.

4. Fill in Data Table 3; calculate work done in filling jug.

DATA TABLE 3		
Water Pressure at Faucet	Volume of Jug	Work Done in Filling Jug
____________ psi ____________ N/m²	____________ gal ____________ cm³ ____________ m³	____________ ft·lb ____________ N·m ____________ J
Recall 1 lb/in = 6895 N/m² 1 liter = 1000 cm³		

ELECTRICAL AND THERMAL SYSTEMS WORK

MATERIALS

Balance.
Small piece of thin, copper wire, approximately 1 1/2 meters long (20 gage or equivalent).
6- or 12-volt storage battery.
Styrofoam plastic cup and block of styrofoam to use as a lid.
Two alligator clip leads for power source.
Voltmeter.
Thermometer (1°C graduations).

PROCEDURE

This experiment will combine electrical and thermal systems in one experiment. Electric potential will be supplied to a small, thin copper wire immersed in a styrofoam cup filled with water. As charge moves through the wire, heat is generated.

1. Measure emf of battery or power source.

2. Measure mass of water in styrofoam cup on balance scale.

3. Record temperature of water in cup before power source is connected.

4. Form copper wire into a coil, being careful not to touch bare wires. Connect clip leads, and immerse coil in water (Figure 8).

5. Connect power source. Keep voltage across wire small enough that no boiling occurs. After temperature has increased about 10 degrees, disconnect all leads, and record temperature.

6. Fill in Data Table 4, calculate heat energy supplied to water (calories) and electrical energy necessary to supply that heat.

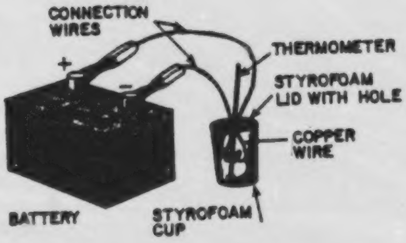


FIG. 8

DATA TABLE 4

Mass of water in cup	____________	g
Beginning temperature	____________	°C
Final temperature	____________	°C
ΔT	____________	°C
Heat energy supplied (Equation 8)	____________	cal
Electrical energy used to supply heat*	____________	J
emf (from voltmeter)	____________	V
Charge transferred (Equation 6)	____________	C
Electrons transferred	____________	electrons

*1 cal = 4.18 J.

**1 C = 6.25×10^{18} electrons.

The student should realize that more electrical energy actually was supplied than needed to heat the water alone. Where is this additional energy consumed?

SELF CHECK

1. Define work and energy, and state the similarities and differences of the two concepts.

2. Define the following units of work and energy, and state the energy systems in which each is used.

   a. Foot-pound.
   b. Newton-meter.
   c. Joule.
   d. Calorie.
   e. British thermal unit.

3. Define the following terms:

   a. Radian.
   b. Current
   c. Specific heat.
   d. Heat capacity.

4. A force of 70N is applied to slide a box a distance of 500 m. How much work is done?

5. 45 ft•lb of work is required to lift an object a distance of 15 ft. How much does the object weigh?

6. A torque of 280 ft•lb does 12 ft•lb of work. What is the angle of rotation in radians and degrees?

7. Explain the two meanings of the English unit ft•lb.

8. How much work is done in removing a nut from a bolt if an average torque of 70 N•m must be supplied for 4 1/2 revolutions? How many ft•lb of work are required?

9. A pump consumes 13,000 J of energy when moving 18 m^3 of water under constant pressure. What pressure (N/m^2 and lb/ft^2) did the pump supply to the water?

10. An air conditioner is capable of providing an air pressure of 1/4 psi. How much energy is required to fill a room 20 ft x 30 ft x 8 ft with cool air?

11. How many calories of heat are required to raise the temperature of 500 g of copper from 29°C to 95°C? (See Table 3.)

12. A household refrigerator may be considered as a "box" of corkboard 2.5 inches thick and 45 square feet of area. Under normal operating conditions (closed door), the interior is maintained at 40°F while the exterior (room temperature) may be around 70°F. If the refrigerator motor runs one-fourth of the time to maintain this temperature difference, at what

rate in Btu/hr must heat energy be withdrawn from the inside of the refrigerator while the motor is running?

13. If a 12-V (d.c.) electric refrigerator consumes 1800 J of energy over a certain period of time to keep food cold, how much charge (coulombs) has been removed?

14. A beaker of water is heated form 25°C to 75°C by the addition of 90 Btu of heat energy. Determine the mass of water heated.

15. Define and give examples of sensible heat and latent heat.

16. State the general equation for work, and explain how it applies in each of the following energy systems:

    a. Mechanical rotational.
    b. Mechanical translational.
    c. Fluidal.
    d. Electrical.
    e. Thermal.

17. A man applies a constant horizontal force to a 5 kg mass in order to accelerate the mass horizontally across a frictionless table. If the man does 20 joules of work to move the mass 3 meters, what is the acceleration of the block?

18. What breaking force applied at the outer edge of a heavy flywheel is required to bring the flywheel to rest in one revolution if the flywheel has a radius of one meter and 600 joules of energy are needed to stop the flywheel?

19. It requires 5000 joules to fill a tank with water from a reservoir located some distance below the tank. What must the effective pressure difference between the top of the tank and the surface of the reservoir be if the tank has a volume of 300 cubic meters?

20. What is the operating voltage of a d.c. electric motor if the motor "consumes" 8000 J of energy in 3 minutes and draws an average current of 3.7 amps?

21. What is the specific heat of a substance if it requires 1500 calories to raise the temperature of 400 grams of the substance 10C°?

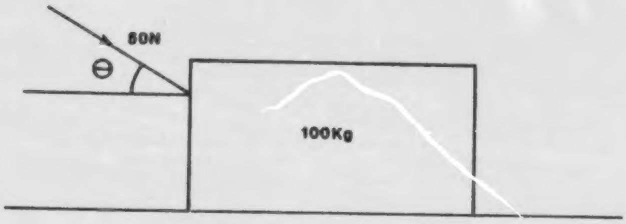
22. How does one explain the fact that it requires 1150 calories to change 10 g of ice at -7°C to water at 32°C? Use 0.5 cal/g°C as the specific heat of ice and show that just using $H = mc\Delta T$ yields a value of thermal energy less than 1150 calories. How is the rest of the heat energy used? Assume no heat is lost to the environment.

23. It requires 10,000 J to charge a battery from 0 to 12 volts. If the current used to charge the battery is 1-amp, how long will it take to charge the battery?

24. A flywheel of radius 1 m is submerged in water and brought to rest in 2 revolutions by a braking force of 100 N applied at the outer edge of the flywheel. Assuming that all of the energy lost by the flywheel goes into heating the water, what will the final temperature of the water be if there is 200 grams of water and the initial temperature of the water was 20°C? Recall that 1 cal = 4.184 J.

25. What is the thermal conductivity of a wall 5 inches thick with an area of 60 ft^2 if the rate of heat energy flow through the wall is 300 Btu/hr when the temperature on one side of the wall is 78°F and the other side of the wall is 28°F?

26. A man applies a 50 N force to a 100 kg mass as shown in the figure below. How does one account for the fact that if he applies the same constant 50 N force to move the mass 5 meters across a frictionless table he does less than 250 joules of work? At what angle θ should the man apply the force to do (a) a maximum amount of work, and (b) a minimum amount of work?



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RATE

CHAPTER 3-0

If a car travels 120 miles in three hours, it travels an average of 40 miles every hour; the car's average speed is 40 miles per hour. Forty miles per hour is referred to as rate of motion or speed of the car. Both distance traveled and time required are used to determine rate of motion. Rate is a quantity that describes how rapidly an occurrence takes place.

This module presents the concept of rate as it applies to mechanical, fluidal, electrical, and thermal systems. In every case, rate expresses the ratio of a displacementlike quantity to an elapsed time:

- In a linear mechanical system, velocity is the ratio of distance traveled to elapsed time.
- In a rotational mechanical system, angular velocity is the ratio of angle of rotation to elapsed time.
- In a fluidal system, volume flow rate is the ratio of volume of fluid moved to elapsed time.
- In an electrical system, current is the ratio of charged transferred to elapsed time.
- In a thermal system, heat flow rate is the ratio of thermal energy transferred to elapsed time.

This module discusses these rates and others and presents the mathematical formulation and units used to express rates in all energy systems.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following rates and, and where applicable, express their basic units both in SI and English systems of units:

Speed and velocity.
Acceleration.
Angular velocity.
Angular acceleration.
Volume flow rate.
Mass flow rate.
Electric current.
Heat flow rate.

- In a linear mechanical system, given all the quantities except one in each of the following groups, determine the unknown quantity:

  Displacement, elapsed time, velocity.
  Initial velocity, final velocity, elapsed time, acceleration.
  Mass, force, acceleration.

- In a rotational mechanical system, given all the quantities except one in each of the following groups, determine the unknown quantity:

  Angular displacement, elapsed time, angular velocity.
  Initial angular velocity, final angular velocity, elapsed time, angular acceleration.

- In a fluidal system, given all the quantities except one in each of the following groups, determine the unknown quantity:

  Volume of fluid moved, elapsed time, volume flow rate.
  Mass of fluid moved, elapsed time, mass flow rate.

- Given two of the following quantities in an electrical system, determine the third:

  Charge transferred.
  Elapsed time.
  Current.

- Given two of the following quantities in a thermal system, determine the third:

  Heat energy transferred.
  Elapsed time.
  Heat flow rate.

- State the general equation for rate, and explain how it applies to each of the following energy systems:

  Mechanical translational.
  Mechanical rotational.

DISCUSSION

Concept Module 1-0, "Force," described forcelike quantities as the physical quantities that cause motion in all energy systems. Concept Module 2-0, "Work," described work done in an energy system in terms of changes that occur in that system as the result of the action of forcelike quantities present. A useful description of the effects of forcelike quantities in a system and work done in that system requires that the time necessary for the changes to occur be considered.

Movement is an occurrence common to every energy system. The rate of movement in each system can be determined by dividing the displacementlike quantity by the time in which the displacement takes place. The unifying formula used for calculation rate is shown in Equation 1.

$$\boxed{\text{RATE} = \frac{\text{DISPLACEMENTLIKE QUANTITY}}{\text{TIME ELAPSED}}} \qquad (1)$$

This module discusses the application of this general rate equation in mechanical, fluidal, electrical, and thermal systems.

MECHANICAL SYSTEMS

MECHANICAL TRANSLATIONAL RATES

The most fundamental rate in a mechanical system is speed. Speed is the rate at which an object moves without regard for the direction of motion of the object. Speed can be calculated by dividing the total length of the path of an object by the time required for the movement along that path. Speed indicates only the rate at which the object moves through some distance; it contains no information concerning the direction of the motion.

Velocity, rather than speed, is used whenever the direction of the moving body is important. Velocity specifies both the speed and direction of the motion. For example, "40 miles per hour" denotes a speed, while "40 miles per hour northeast" denotes a velocity.

When an object moves in a straight line, it is said to be in linear or translational motion. The object may stop or reverse direction, but its motion always lies along the same straight line. This discussion of mechanical translational rates will deal only with cases of straight-line motion. The term "speed" will be used when the direction of motion is not important; velocity will be used when both speed and direction are important. The formula for calculating speed (the magnitude of velocity) in a linear mechanical system is given by Equation 2.

$$v = \frac{d}{t} \qquad (2)$$

where

v = speed (magnitude of velocity).

d = displacement.

t = elapsed time.

The symbol v commonly is used for both speed and velocity. If the quantity identified as v includes a direction, it is the vector quantity velocity. If it does not include a direction, it is the scalar quantity speed.

Example A illustrates how Equation 2 is used to solve a problem.

EXAMPLE A: SPEED AND VELOCITY OF AN AUTOMOBILE

Given: An automobile travels 250 feet east in 10 seconds.

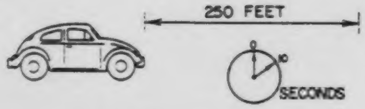
Find: Its speed and velocity.

Solution: Speed:

$$v = \frac{d}{t}$$

$$= \frac{250\text{ ft}}{10\text{ s}}$$

$$v = 25\ \frac{\text{ft}}{\text{s}}$$



The speed of the automobile is 25 ft/s. If direction is specified, we have a velocity.

Velocity = 25 ft/s, east.

If direction of the moving car in Example A were given as west, the car's velocity would be 25 ft/s, west.

Distance (displacement) often is measured in units such as feet, miles, or meters and time in units such as seconds, minutes, or hours. Common units for speed are summarized in Table 1.

TABLE 1: COMMON UNITS FOR SPEED

System of Measure	Common Units for Speed
English	ft/s or miles/hour (44 ft/s = 30 mi/h)
SI	m/s or kilometers/hour (0.278 m/s = 1 km/h)

Example B illustrates the use of Equation 2 to determine elapsed time if speed and distance are known.

EXAMPLE B: TIME TO RUN A MILE

Given: A runner can average 13.34 mi/h.

Find: Time required to run one mile.

Solution: Speed $v = 13.34$ mi/h

$$= (13.34\ \cancel{\text{mi/h}})\left(\frac{44\ \text{ft/s}}{30\ \cancel{\text{mi/h}}}\right)$$

$$v = 19.56\ \text{ft/s}$$

From Equation 2: $t = \frac{d}{v}$

$$= \frac{5280\ \text{ft}}{19.56\ \text{ft/s}}$$

$$= 270\ \text{s}$$

$$= (270\ \text{s})\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$

$$t = 4.5\ \text{min}$$

Both Examples A and B involved average speed of an object. Speed was treated as if it were constant through the entire measured time. However, in many cases the speed of a moving object fluctuates as it moves in a fixed direction. The rate of change of linear speed is called "linear acceleration." A constant linear acceleration is calculated by using the relationship shown in Equation 3.

$$a = \frac{v_2 - v_1}{t} \qquad (3)$$

where

a = linear acceleration.
v_1 = initial speed (speed before change).
v_2 = final speed (speed after change).
t = time in which change takes place.

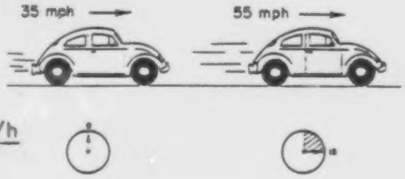
The units of acceleration are velocity units (m/s, ft/s, mi/h) divided by time units. The most commonly-used acceleration units are ft/s^2 and m/s^2. An acceleration of 1 m/s^2 means that the velocity increases by 1 m/s in 1 s.

Example C shows the use of Equation 3 in determining acceleration.

EXAMPLE C: ACCELERATION OF A CAR

Given: A driver increases the speed of a car uniformly from 35 miles per hour to 55 miles per hour in 15 seconds.

Find: Acceleration of the car.



Solution:

$$a = \frac{v_2 - v_1}{t}$$

$$= \frac{55 \text{ mi/h} - 35 \text{ mi/h}}{15 \text{ s}}$$

$$= \frac{20 \text{ mi/h}}{15 \text{ s}}$$

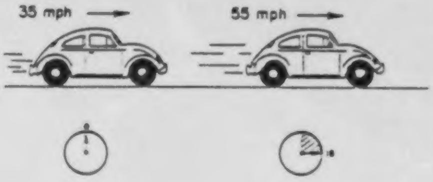
$$a = 1.33 \, \frac{\text{mi/h}}{\text{s}}$$

The car was accelerated 1.33 mi/h every second.

Units shown in Example C may look strange because two different time units appear. The units often are reduced in order that only one time unit is used. To reduce units in Example C in order that only one time unit is used, the unit for speed is converted from miles per hour to feet per second. Example D shows this alternate solution to Example C.

EXAMPLE D: ALTERNATE SOLUTION FOR EXAMPLE C

Given: A driver increases the speed of a car uniformly from 35 miles per hour to 55 miles per hour in 15 seconds.



Find: Acceleration of the car.

Solution:

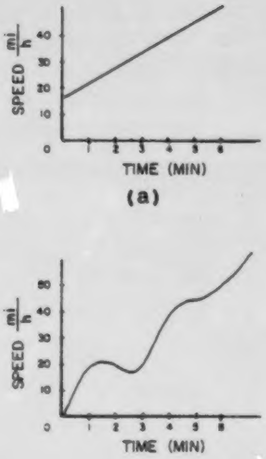
$$a = \frac{v_2 - v_1}{t}$$

$$= \frac{(55 - 35)\ \cancel{mi/h}\ \left(\frac{44\ ft/s}{30\ \cancel{mi/h}}\right)}{15\ s}$$

$$= \frac{29.3\ ft/s}{15\ s}$$

$$a = 1.96\ \frac{ft/s}{s}$$

The units $\frac{ft}{s} \div s = \frac{ft}{s^2}$



(a)

(b)

FIG. 1

Acceleration is illustrated better by the graphs in Figure 1, where speed is recorded on the vertical axis, and where time is recorded on the horizontal axis.

A uniform change in speed, indicated on the graph by a straight line, means that linear acceleration is constant. A nonuniform change in speed, indicated on the graph by a curving line, means that linear acceleration is varying. The concepts of speed and acceleration are illustrated further in Figure 2, which represents the changing speed of a car during a short trip.

Section A of the graph shows acceleration of the car as it starts from rest and increases its speed to approximately 10 mi/h. Since the graph rises very rapidly, rate of

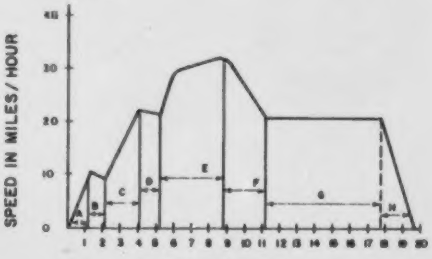


FIG. 2

increase in speed (acceleration) is large. The car in in low gear. Section B represents a slowing down (deceleration) of the car as its gears are shifted from low to second. Section C represents acceleration as the car increases its speed while in second gear. Section D again represents deceleration as gears are shifted from second to third. Section E at first represents an increasing speed and thus an acceleration. But notice that acceleration during Section E does not remain constant. The graph shows clearly that, after the initial high acceleration, the car accelerates less rapidly. Section F shows a rapid deceleration, which must result from a braking action on the part of the driver. Section G shows no acceleration, because the speed does not change during this time. Section H represents a final deceleration, which brings the car to a stop.

Figure 2 indicates clearly both accelerations and decelerations. "Acceleration" is the term generally applied to a rate of increase of speed, and "deceleration" is the term applied to a rate of decrease of speed. If a moving object is decelerating, the value for change of speed in Equation 3 will be negative, giving an overall negative value for acceleration. Thus, deceleration also is called "negative acceleration."

Acceleration is produced only when an unbalanced force sufficient to cause a change in rate of motion is applied to an object. Amount of acceleration produced depends upon both magnitude of the unbalanced force and mass of the object. The greater the unbalanced force on a given object, the greater the acceleration will be. Also, the more massive an object, the harder it is to accelerate. Furthermore, the object always will accelerate in the direction of the unbalanced force. These physical relationships (referred to as Newton's second law of motion) are stated in Equation 4.

$$a = \frac{F}{m} \tag{4}$$

where

a = acceleration.
F = unbalanced force.
m = mass of object.

This equation often is stated in the form —

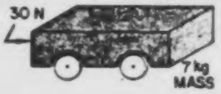
$$F = ma$$

To apply this equation, force and mass must be used in correct units. For example, when mass is measured in kilograms and force is measured in newtons ($\frac{kg \cdot m}{s^2}$), acceleration will be in m/s^2. Use of the acceleration formula is illustrated in Examples E and F.

EXAMPLE E: ACCELERATION OF A MASS

Given: An unbalanced force of 35 N is applied to a 7-kg object.

Find: Acceleration produced (neglect friction).



Solution:

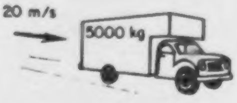
$$a = \frac{F}{m}$$

$$= \frac{35\ \cancel{N}}{7\ kg} = \frac{35\ \frac{\cancel{kg}\cdot m}{s^2}}{7\ \cancel{kg}}$$

$$a = 5\ m/s^2$$

EXAMPLE F: TRUCK DECELERATION

Given: A 5000-kg truck has a speed of 20 m/s.



Find: Braking force required to stop in 60 seconds.

Solution: First, find rate of deceleration:

$$a = \frac{v_2 - v_1}{t}$$

$$= \frac{0 - 20\ m/s}{60\ s}$$

$$= \frac{-20\ m/s\ (1/s)}{60\ s/1\ (1/s)}$$

$$a = -0.33\ m/s^2$$

Now, find force required:

$$F = ma$$

$$= 5000\ kg \cdot -0.33\ m/s^2$$

$$F = -1650\ \frac{kg\cdot m}{s^2} = -1650\ N$$

A force of 1650 N is required to stop the truck in the time indicated. The negative sign tells that braking force was applied in a direction opposite to motion of the truck.

Acceleration of a body due to earth's gravity is 32 ft/s^2 or 9.8 m/s^2. This means that a freely-falling body is increasing its velocity an additional 32 ft/s for every second that it falls (assuming that there is no wind resistance). After two seconds, a falling body that started from rest will be falling with a velocity of 64 ft/s, and after five seconds, with a velocity of 160 ft/s. Weight of a body is a measure of the pull of gravity of the mass of that body. This relationship is shown in Equation 5.

$$W = mg \tag{5}$$

where

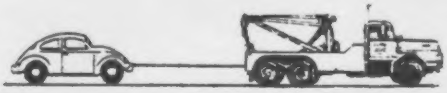
W = weight (N or lb).
m = mass (kg or slugs).
g = acceleration of gravity.

The importance of using weight and mass correctly is shown by Example G.

EXAMPLE G: TOWING AN AUTOMOBILE

Given: A 3700-lb automobile is being towed on a level road by a towtruck accelerating at 3.8 ft/s^2.

Find: Tension in the towline (force required).



Solution: F = ma

Wrong Solution:

$$F \text{ (tension)} = (3700 \text{ lb})(3.8 \text{ ft/s}^2)$$

$$F = 14{,}060 \ \frac{\text{lb}\cdot\text{ft}}{\text{s}^2}$$

A tension of $\frac{\text{lb}\cdot\text{ft}}{\text{s}^2}$ does not make sense. Tension should be in pounds! What went wrong?

The mistake is that 3700 lb is not the mass but the weight of the car. Therefore, this solution is incorrect.

Correction Solution:

First, find mass of the car being accelerated:

$$W = mg$$

$$m = \frac{W}{g}$$

$$m = \frac{3700\ \text{lb}}{32\ \text{ft/s}^2}$$

Recall that

$$m = 115.6\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}} = 115.6\ \text{slugs} \qquad \frac{\text{lb}\cdot\text{s}^2}{\text{ft}} = \text{slugs}$$

Now, find tension on the rope:

$$\begin{aligned} F &= ma \\ &= (115.6\ \text{slugs})(3.8\ \text{ft/s}^2) \\ &= \left(115.6\ \frac{\text{lb}\cdot\cancel{\text{s}^2}}{\cancel{\text{ft}}}\right)\left(3.8\ \cancel{\text{ft/s}^2}\right) \\ F &= 439\ \text{lb} \end{aligned}$$

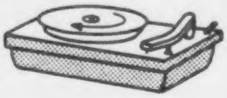
Tension in the towline is 439 lb.

MECHANICAL ROTATIONAL RATES

Rate of rotational motion is called angular velocity. Angular velocity is defined as the rate of change of an angle with time. Common units for measuring angular velocity are revolutions per minute (rpm or r/min) and radians per second (rad/s).

Recall from Module 2-0, "Work," that 1 revolution equals 2π radians. The relationship between rpm and rad/s is illustrated in Example H.

EXAMPLE H: UNITS FOR ANGULAR VELOCITY



Given: A phonograph turntable has an angular velocity of 45 rpm.

Find: The angular velocity in rad/s.

Solution: Angular velocity $= (45\ \cancel{\text{r}/\text{min}})\left(\frac{2\pi\ \text{rad}}{1\ \cancel{\text{r}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\ \text{s}}\right)$

Angular velocity - 4.7 rad/s.

The angular velocity is 4.7 rad/s.

In the rotational mechanical system, the basic rate equation is Equation 6.

$$\omega = \frac{\theta}{t} \tag{6}$$

where
ω = angular velocity.
θ = angular displacement.
t = elapsed time.

The use of this equation in determining angular velocity is illustrated in Example I.

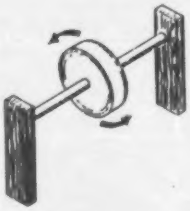
EXAMPLE I: ANGULAR VELOCITY

Given: A grindstone rotates 600 revolutions in 5 minutes.

Find: Angular velocity.

Solution:

$$\begin{aligned}\omega &= \frac{\theta}{t}\\ &= \frac{600\ \text{revolutions}}{5\ \text{minutes}}\\ &= 120\ \text{rpm}\\ &= 120\left(\frac{\cancel{\text{rev}}}{\cancel{\text{min}}}\right)\left(\frac{2\pi\ \text{rad}}{\cancel{\text{rev}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\ \text{s}}\right)\\ &= 120\ \frac{2\pi\ \text{rad}}{60\ \text{s}}\\ &= 4\pi\ \text{rad/s}\\ \omega &= 12.56\ \text{rad/s}\end{aligned}$$



When an unbalanced torque is applied to a rotary system, the angular velocity of that system is changed. The rate at which this change takes place is called "angular acceleration." Angular acceleration can be determined by dividing change in angular velocity by time required for this change. The formula for calculating angular acceleration is given in Equation 7.

$$\alpha = \frac{\omega_2 - \omega_1}{t} \qquad (7)$$

where
α = angular acceleration.
ω_1 = initial angular velocity (before change).
ω_2 = final angular velocity (after change).
t = time in which the change takes place.

Example J shows how the formula can be used to solve a problem.

EXAMPLE J: ANGULAR ACCELERATION

Given: A brake is applied to a flywheel for 5 seconds, reducing angular velocity of the flywheel from 2.2 rad/s to 1.8 rad/s.

Find: Angular acceleration produced.

Solution:

$$\alpha = \frac{\omega_2 - \omega_1}{t}$$

$$= \frac{1.8 \text{ rad/s} - 2.2 \text{ rad/s}}{5 \text{ s}}$$

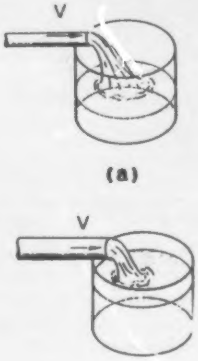
$$= \frac{-0.4 \text{ rad/s}}{5 \text{ s}}$$

$$\alpha = -0.08 \text{ rad/s}^2$$

Angular acceleration produced is -0.08 rad/s^2. The negative sign indicates this is a deceleration of 0.08 rad/s^2.

FLUIDAL SYSTEMS

The rate of flow of fluids is often an important consideration in hydraulic and pneumatic systems. However, a measure of fluid velocity is not in itself adequate to describe flow rate, since the volume or capacity of fluid moving through a duct or pipe must be considered. Thus, one measures volume per second, rather than simply distance per second. For example, if the velocity of fluid in two pipes is the same, less fluid will flow through a small pipe in the same time, as illustrated in Figure 3. The volume of fluid that flows in a given time is much more important. This quantity is called the "volume flow rate." It is found by dividing a measured volume by the time required for this volume to flow from a pipe. Equation 8 shows the formula for calculating volume flow rate. Note that the displacementlike quantity in this instance becomes the volume of fluid moving past a certain position in the elapsed time.



(a)

(b)

FIG. 3

$$Q_V = \frac{V}{t} \qquad (8)$$

where

Q_V = volume flow rate.
V = volume of fluid displaced.
t = elapsed time.

Units of volume flow rate could be any volume unit divided by any time unit. Examples include the following: m^3/s, ft^3/s, gal/min, liters/s, cm^3/s. Example K illustrates how volume flow rate is calculated.

EXAMPLE K: VOLUME FLOW RATE

Given: A swimming pool holds 10,000 gallons; 2 hours are needed to fill pool.

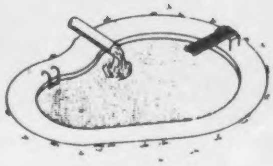
Find: Volume flow rate of water through the hose.

Solution:

$$Q_V = \frac{V}{t}$$

$$= \frac{10,000 \text{ gal}}{(2\text{h})\left(\frac{60 \text{ min}}{1 \text{ h}}\right)}$$

$$Q_V = 83 \text{ gal/min}$$



For some purposes (especially for gases), the desirability arises to consider mass flow rate rather than volume flow rate. Mass flow rate is defined as "the fluid mass moving past a given position in a given amount of time." Equation 9 defines mass flow rate. Note that here the displacementlike quantity becomes the mass moving past a fixed position in the elapsed time.

$$Q_m = \frac{m}{t} \tag{9}$$

where

Q_m = mass flow rate.
m = mass of fluid displaced.
t = elapsed time.

Example L shows how flow rate can be calculated.

EXAMPLE L: MASS FLOW RATE

Given: 180 kg of ammonia gas flowing through the cooling coils of a refrigerated truck in 6 minutes.

Find: Mass flow rate of the gas.

Solution:

$$Q_m = \frac{m}{t}$$

$$= \frac{180,000 \text{ g}}{6 \text{ min}}$$

Q_m = 30,000 g/min = 30 kg/min

The ammonia gas is flowing at a rate of 30,000 grams/minute of 3 kg/min.

ELECTRICAL SYSTEMS

In electrical systems the displacementlike quantity is charge. Recall from Module 1-0, "Force," that like charges attract and unlike charges repel. This characteristic of electrical charge results in a tendency for positive and negative charges to be distributed as evenly as possible in an electrical system. If the system contains a concentration of positive charge in one volume and a concentration of negative charge in another, a potential difference (forcelike quantity) is present. If charges are free to move, the potential difference will cause charge to flow until the system is in electrical equilibrium. Electric current is the rate of charge flow and is defined by Equation 10.

$$I = \frac{Q}{t} \qquad (10)$$

where

I = electric current.
Q = charge transferred.
t = elapsed time.

Current is measured in amperes (amp or A). One ampere is equal to one coulomb of charge flowing each second. In metallic electrical conductors the moving charge is composed of electrons that flow through the metal. There are 6.25×10^{18} electrons in one coulomb. Thus, a current of one amp represents a transfer of 6.25×10^{8} electrons from the point of negative potential to the point of positive potential every second. Example M illustrates the use of Equation 10 in determining the charge transferred in a household circuit.

EXAMPLE M: ELECTRICAL CURREN

Given: A home electrical circuit is limited to 15 amps by a fuse or circuit breaker.

Find: How many electrons have flowed through the circuit in 10 seconds when it has a maximum load.

Solution: A current of 15 amps is equivalent to a flow rate of

15 coulombs/seconds.

$$I = \frac{Q}{t}$$

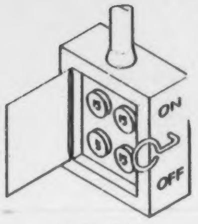
$$Q = It$$

$$= (15\ C/s)(10\ s)$$

$$= 150\ C$$

$$= (150\ C)\left(\frac{6.25 \times 10^{18}\ \text{electrons}}{1\ C}\right)$$

$$Q = 9.4 \times 10^{20}\ \text{electrons}$$



FUSE BOX

Alternating current (a.c.) and direct current (d.c.) are terms that refer to the direction in which electrical charge moves. Direct current means that electrical charge flows in one direction only, even though the amount of current may vary. In alternating current, the direction of the current changes (alternates) so that the charge flows back and forth. Commercial power companies generate electric current that changes direction at the rate of 120 times each second. (The popularity of a.c. arises from its ability to transmit electrical energy great distances without large heat losses in the wires. One advantage in the use of a.c. electricity in our homes is that a smaller wire will carry more current.)

THERMAL SYSTEMS

In any system where a temperature difference exists, heat moves from the high-temperature region to the low-temperature region. Movement in this system, however, is different from movement in the other energy systems discussed in this module. In other systems, an object of some kind moves from one location to another; in thermal systems, the quantity that moves is heat energy. This energy is contained in a substance in the form of vibrational energy of the molecules. A higher temperature indicates a higher vibrational rate of the molecules and thus a greater quantity of heat energy contained in the substance.

In any substance, heat energy is transferred when an energetic vibrating molecule bumps a neighboring molecule with a lower vibrational rate. The more-energetic molecules loses some of its thermal energy and its vibrational rate decreases; the less-energetic molecule gains this energy and its vibrational rate increases. This process continues in a closed system until the thermal energy is evenly distributed and there is no longer a temperature difference.

Heat flow rate is the amount of heat energy flowing through a surface in a given amount of time. The formula used to calculate heat flow rate is given in Equation 11. Here the displacementlike quantity is less clear than distance, volume or mass. In thermal systems, it becomes a measure of the heat energy flow past a fixed position in the time elapsed.

$$Q_H = \frac{H}{t} \tag{11}$$

where
Q_H = heat flow rate.
H = heat energy transferred.
t = elapsed time.

Designers of heating and cooling systems are vitally concerned with rate of heat flow. Manufacturers' specifications for heating and cooling systems, however, sometimes are misleading, because they are given in units of Btu's. This terminology is misleading, because what the manufacturer means by "Btu's" is actually "Btu's per hour," which is a heat flow rate. For instance, a 10,000-Btu air conditioner is capable of removing room heat at a rate of 10,000 Btu's every hour. Example N shows how Equation 11 is used to solve problems involving heat flow rate.

EXAMPLE N: HEAT FLOW RATE

Given: A 5000-Btu tent heater runs continuously for 5 hours and 30 minutes.

Find: Amount of energy produced.

Solution: The 5000-Btu specification means a heat flow rate of 5000 Btu/hour.

$$Q_H = \frac{H}{t}$$

$$H = Q_H t$$

$$= (5000\ \text{Btu/h})(5\ 1/2\ \text{h})$$

$$H = 27{,}500\ \text{Btu}$$

SUMMARY

The concept of rate is important in all energy systems. Any change in a system takes place in a certain elapsed time. Rate is the ratio of change in a physical quantity to the time required for that change. This module has dealt with the basic rates in four energy systems — mechanical, fluidal, electrical, and thermal. Many other rates commonly are used in science and technology. Some of these are discussed in application modules. One rate is so important that it is considered to be a separate unifying concept. Power is the rate of energy transfer and will be discussed in detail in Module 6-0, "Power."

Table 2 summarizes the basic rates presented in this module.

TABLE 2: RATE AS A UNIFYING CONCEPT

ENERGY SYSTEM	$\text{RATE} = \frac{\text{DISPLACEMENTLIKE QUANTITY}}{\text{ELAPSED TIME}}$
MECHANICAL	
Translational	$\text{Velocity} = \frac{\text{Displacement}}{\text{Elapsed Time}}$
Rotational	$\text{Angular Velocity} = \frac{\text{Angular Displacement}}{\text{Elapsed Time}}$
FLUIDAL	$\text{Volume Flow Rate} = \frac{\text{Volume Displaced}}{\text{Elapsed Time}}$
	$\text{Mass Flow Rate} = \frac{\text{Mass Displaced}}{\text{Elapsed Time}}$
ELECTRICAL	$\text{Current} = \frac{\text{Charge Transferred}}{\text{Elapsed Time}}$
THERMAL	$\text{Heat Flow Rate} = \frac{\text{Heat Energy Transferred}}{\text{Elapsed Time}}$

MECHANICAL SYSTEMS

MATERIALS

Toy car, either wind-up or battery powered.
2-meter track for car.

PROCEDURE

The student is to use a wind-up or a battery powered toy car to calculate velocity of the car and speed with which the wheels turn.

1. Mark "start" and "finish" lines on a straight course at least two meters long.
2. Start car a reasonable distance in front of "start" line (to allow speed to become steady); time car carefully as it travels from "start" to "finish" line. Record this time in Data Table 1.
3. Calculate average speed of car in m/s, ft/s and mi/h.
4. Measure wheel diameter on car and list in Data Table 1.
5. Calculate wheel circumference, $c = \pi d$.
6. Divide raceway length by wheel circumference to determine how many times wheel rotated while it traveled across raceway.
7. Calculate angular velocity of wheel in rpm and in rad/s.

DATA TABLE 1

Raceway length ____________

Time from start to finish ____________

Average speed (calculated) ____________ m/s

____________ ft/s

____________ mi/h

Wheel diameter ____________ mm

Wheel circumference ($c = \pi d$) ____________ mm

Number of rotations to cover raceway ____________ revolutions

Angular velocity of wheel (calculated) ____________ rpm

____________ rad/s

FLUID SYSTEMS

MATERIALS

Gallon or liter container.
Meter stick.
Stopwatch or timer.
Balance.

PROCEDURE

Several cooling systems require a specified volume flow rate for adequate operation. Specified values may be given in units of gallons per minute or liters per minute. In this experiment, a water faucet is used to simulate a discharge pipe from a cooling system and to calculate volume flow rate coming from the faucet (Figure 4).

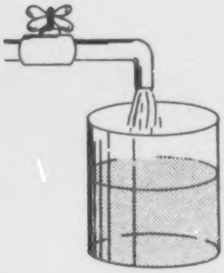


FIG. 4

1. Collect flowing water in a container such as a gallon or liter container; accurately measure time required to fill container.
2. Record values in Data Table 2. From these measurements, calculate volume flow rate in both gallons per minute and in liters per minute. Enter values in Data Table 2.
3. Using a balance scale, determine mass of water in container.
4. Calculate mass flow rate in kilograms per minute and in slugs per minute. Enter values in Data Table 2.

DATA TABLE 2

Container volume ____________

Time to fill container ____________

Mass of water in container ____________

Volume flow rate ____________ gal/min

____________ l/min

Mass flow rate ____________ kg/min

____________ slugs/min

THERMAL SYSTEMS

MATERIALS

Conductometer (Sargent Welch #1653 or equivalent).
Bunsen burner or candle.
Paraffin.

PROCEDURE

Heat flows more rapidly through some materials than through others. This experiment determines which of the materials tested allows the greatest heat flow rate. The instrument used to make these determinations is the "Conductometer," a device having several spokes radiating outward from the central hub, as illustrated in Figure 5.

1. Place a piece of melted paraffin on end of each spoke; allow it to cool.

2. Place a lighted bunsen burner (or candle) under central hub. Observe which spokes have most rapid head flow by noting the order in which paraffin pieces melt. List the metals in order, from most rapid head flow rate to slowest heat flow rate. (1) __________ (2) ___________ (3) __________ (4) ___________ (5) __________ (6) _________.

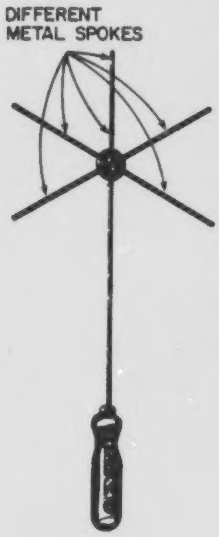


FIG. 5

ELECTRICAL SYSTEMS

MATERIALS

6- or 12-volt battery or d.c. power supply.
6- or 12-volt d.c. bulb with mounting base and connecting wires.
d.c. ammeter 0- to 2-amp.

PROCEDURE

Rate of flow of electric charge (electric current) is measured with an ammeter. Place ammeter in an electric circuit along with a battery and a small d.c. light bulb, according to Figure 6. How much electric current is recorded by milliammeter when the bulb is lit?

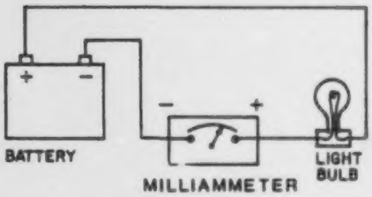


FIG. 6

Current = ____________.

How many coulombs/s is this? __________.

If the light bulb burns for one hour, how many electrons have moved through bulb filament?

SELF CHECK

1. Define the following rates, and express their basic units both in the SI and the English system of units:

   Linear speed.
   Linear acceleration.
   Angular velocity.
   Angular acceleration.
   Volume flow rate.
   Mass flow rate.
   Electrical current.
   Heat flow rate.

2. Calculate an airplane's average speed if it travels 3200 kilometers in 3.5 hours.

3. If a car traveling at 55 miles per hour stops in 16.3 seconds, calculate the car's deceleration in ft/s^2.

4. If a driver averages 55 mi/h, how much time will be required to make a 250-mile trip?

5. A force of 200 N accelerates an object with a mass of 100 kg. If the object was initially at rest, determine its speed after three seconds.

6. A flywheel of radius 60 cm initially at rest requires 25 seconds to reach an angular velocity of 1750 r/min. Calculate angular acceleration of the flywheel.

7. A motor rotates at the rate of 5000 rpm. How much time is required for one complete revolution?

8. A gas pump moves gasoline from an underground storage tank to the tank in a car at the rate of 15 liters per minute. Calculate time required to fill a 20-gallon gas tank with this pump.

9. If an air conditioner is rated at 10,000 Btus, calculate how much heat energy it will remove from a room in one day if it operates 20% of the time.

10. Electric current flows at the rate of 0.5 amps through a normal 60-watt light bulb. Calculate how many coulombs of electric charge this represents in 12 hours of continuous operation.

11. State the general equation for rate; and explain how it applies to mechanical, fluidal, electrical, and thermal energy systems.

12. A crane lifts a 2000 kg mass upwards. If the tension in the cable is 20,000 N, what is the acceleration of the block? (Hint, when using F = ma, F is the net force on the body.)

13. How long does it take an object to accelerate from 3 m/s to 15 m/s if the object has a mass of 12 kg and is accelerated by a force of 20 N?

14. What is the initial angular velocity of a flywheel if, after two minutes, the flywheel is rotating clockwise at 250 rad/s and the acceleration of the flywheel is a constant 4 rad/s^2, clockwise?

15. An engine draws a current of 6 amps for 15 minutes. If the engine is 100% efficient and operates on 12 V, how much work does the engine do in this time?

16. (a) At what rate must a pump move water into a swimming pool if the pool has a capacity of 3000 gallons and is to be filled in 90 minutes?

    (b) If one gallon of water has a volume of 231 in^3, and water has a weight density of 62.4 pounds/ft^3, what average horsepower is the pump generating if 1 hp = 550 foot pounds/s and the water must be raised through an average height of 10 feet?

17. Through what angular distance does a flywheel turn during the time it takes the flywheel to accelerate from 10 rad/s to 100 rad/s if the flywheel undergoes a constant acceleration of 14 rad/s^2?

18. If the linear velocity of a point on a flywheel is V, then the angular velocity of that point is V/r, where r is the radius out to that point. Suppose a car accelerates around a curve at 100 mi/h^2, and just before entering the curve the car's speed is 30 mi/h. What is the angular speed of the car as it leaves the curve if it takes the car 10 seconds to move through the length of the curve? The curve has a radius of 50 ft.

19. If the acceleration is constant, the V in equation (2) of the module can be replaced by the average velocity. If a block of mass 1 kg undergoes a constant acceleration and, starting from rest, travels 10 meters in 1 second, find: (a) the speed of the block at the end of the first second, (b) the acceleration of the block, and (c) the force necessary to produce this acceleration.

20. What is the mass flow rate in kg/s through a hose if 20 ℓ of water flow out of the end of the hose in 10 seconds? The density of water is 1000 kg/m^3.

21. How long will it take a car traveling 50 m/s to overtake a car traveling 20 m/s if the first car is initially 2000 meters behind the second car?

22. What is the current in amperes flowing through a wire if 50×10^{17} electrons flow through the wire in 20 seconds?

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MOMENTUM

CHAPTER 4-0

Moving objects possess a very important physical quantity called "momentum." "Linear momentum," a vector quantity, depends upon the mass and the linear velocity of the object. "Angular momentum," also a vector quantity, depends upon the moment of inertia and the angular velocity of a rotating mass.

This module explains the mathematical relationships involved with both linear and angular momentum. Elastic and inelastic collisions are considered, as well as "the law of conservation of momentum."

The law of conservation of momentum is an important unifying principle in physics. Whenever momentum is transferred from one object to another, or from one system to another, total momentum always is conserved. The concept of momentum finds the greatest practical application in mechanical and fluid systems.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following terms; state the appropriate units in the mk/s system (SI) and the cg/s system; and give the equation for each:

  Linear momentum.
  Angular momentum.
  Impulse.
  Angular impulse.
  Moment of inertia.

- Given two of the following quantities, determine the third:

  Mass of an object.
  Velocity of the object.
  Momentum of the object.

- Given two of the following quantities, determine the third:

    Moment of inertia of an object.
    Angular velocity of the object.
    Angular momentum of the object.

- Given all the following quantities except one describing a linear collision, determine the unknown quantity:

    Mass of first object.
    Initial velocity of first object.
    Final velocity of first object.
    Mass of second object.
    Initial velocity of second object.
    Final velocity of second object.

- Explain the following concepts in a short paragraph each:

    Conservation of linear momentum.
    Conservation of angular momentum.

- Use a given equation to calculate the force produced on one blade of a reaction turbine, given the velocity of fluid and the mass of fluid per unit time striking the blade.

DISCUSSION

A large ocean liner has a tremendous mass. When such a liner approaches a dock (Figure 1), the crew reduces the ship's velocity to a very low value. As the enormous ship contacts the dock, a huge force must be absorbed by the dock in order to stop the ship's motion completely. Very massive objects are difficult to stop.

FIG. 1

A bullet fired from a gun at high velocity has a tremendous impact on a target, although the mass of the bullet may be only a few grams. The huge force exerted by the bullet upon the target can be seen by the destructive power of even a small-caliber bullet fired at a glass bottle (Figure 2). If the bottle is filled with water, the velocity of the bullet is reduced more upon impact; and the destruction of the target is even greater. Rapidly-moving objects are difficult to stop.

FIG. 2

These examples illustrate the concept of momentum. Momentum is an important physical quantity related to the motion of objects. Either a small object moving with a high velocity, or a large object moving with a low velocity, possesses relatively large amounts of momentum and exerts large forces on other objects when stopped. Both mass and velocity are necessary in determining the momentum of a body.

Considerations of momentum for moving objects is divided into two classifications:

(1) Linear momentum if the object is moving in a straight line.
(2) Angular momentum if the object is rotating.

LINEAR MOMENTUM

Linear momentum of a single object of mass moving with velocity v is defined by Equation 1.

$$\vec{p} = m\vec{v} \qquad (1)$$

where
p = linear momentum.
m = mass.
v = velocity.

(The arrow drawn over p and v simply indicates that they are vectors.)

Since velocity is a vector quantity (one that has both magnitude and direction), momentum also must be a vector quantity if Equation 1 is to hold true. The direction of the momentum vector is the same as that of the velocity vector, but its magnitude and units are different from that of the velocity vector. The vector nature of momentum will be considered more thoroughly later in this module. As indicated in Equation 1, arrows always are drawn over vector quantities when the direction, as well as the magnitude, is important in the problem. In momentum problems, both magnitude and direction are important.

Units for momentum are consistent with Equation 1. If mk/s units are used for mass (kg) and velocity (m/s), then the momentum units obtained would be (kg-m/s). However, if the cg/s system is used, mass in grams times velocity in cm/s yields momentum in units of g-cm/s. Examples A and B illustrate the calculation of momentum for two greatly-different masses.

EXAMPLE A: LINEAR MOMENTUM OF LARGE MASS

Given: An automobile whose mass is 2500 kg is moving with a velocity of 70 km/h. (Assume that the car is moving in a straight line.)

Find: Its momentum.

Solution: The momentum of the automobile will be in the same direction as its velocity.

$$\vec{p} = m\vec{v}$$

$$= (2500\ \text{kg})\left(\frac{70{,}000\ \text{m}}{\text{h}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)$$

$$\vec{p} = 48{,}600\ \text{kg-m/s}$$

EXAMPLE B: LINEAR MOMENTUM OF SMALL MASS

Given: An electron whose mass is 9.1×10^{-28} grams is moving in an electric wire at an effective velocity of approximately 0.1 cm/s.

Find: Its momentum.

Solution:

$$\vec{p} = m\vec{v}$$

$$= (9.1 \times 10^{-23}\ \text{g})(1 \times 10^{-1}\ \text{cm/s})$$

$$\vec{p} = 9.1 \times 10^{-29}\ \text{g-m/s}\quad \text{(a small momentum!)}$$

IMPULSE AND MOMENTUM CHANGE

When force is applied to a moving object, velocity change is produced. The velocity either may increase or decrease, depending upon the direction in which the force is applied to the object. The change in velocity requires a corresponding change in momentum of the object, because momentum depends upon the velocity, as indicated in Equation 1. The amount of change in momentum is equal to the product of the force and the elapsed time during which the force acts. This relationship is expressed by Equation 2.

$$\Delta p = F\Delta t \qquad (2)$$

where

Δp = change in momentum.
F = force.
Δt = elapsed time during which the force acts.

The term applied to the product of force and time is "impulse."

The concept of impulse and momentum change is understood better when analyzed with the following short derivation:

$$F = ma \text{ (Newton's second law)}$$

$$F = m\frac{\Delta v}{\Delta t} \text{ (since } a = \frac{\Delta v}{\Delta t} = \frac{\text{change in velocity}}{\text{time elapsed}}\text{)}$$

$$F\Delta t = m\frac{\Delta v}{\Delta t}\Delta t \text{ (multiplying both sides by } \Delta t\text{)}$$

$$F\Delta t = m\Delta v$$

$$\text{Impulse} = \text{change in momentum}$$

From this derivation, impulse is equivalent to the change in momentum produced when a force is applied to an object. Consider the case of a baseball being struck by a bat. The baseball is given its initial momentum and velocity by the pitcher. The direction of these quantities is, of course, toward the batter. As the bat contacts the ball, an enormous force is applied to the ball for the short time during which the two objects are in contact. This force is large enough to cause a considerable change in the momentum and velocity of the ball. The direction of the ball is changed, as well as is its speed. The impulse, or the product of the force applied and the time during which the bat and the ball were in contact, is equal to the change in momentum of the ball, provided the vector nature of momentum is considered.

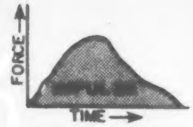


FIG. 3

In most "réal-life" examples of impulse and changes in momentum, the amount of force varies during the time of contact; however, "average" force usually can be determined in order to complete the necessary calculations. For example, the graph in Figure 3 represents the variation of the force applied to the ball during the contact time.

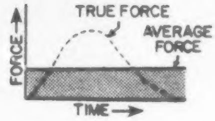


FIG. 4

The shaded area under the curve in Figure 3 is equal to the impulse. The irregular shape of this curve makes an exact calculation of impulse very difficult. Instead, an average constant force can be found, as in Figure 4. If the average force, represented by the straight line in Figure 4, were applied continually to the ball throughout the same contact time, the same impulse would be produced. This is true, because the shaded areas under the curves in Figures 3 and 4 (i.e., impulse) are equal. The shaded areas are made equal, of course, by use of the correct average force. Average force is determined in this manner.

CONSERVATION OF MOMENTUM

The total momentum of an isolated system does not change; it is conserved. An isolated system is one in which there are no external, unbalanced forces acting upon the system. For example, the forward momentum of a baseball does not change noticeably between the time that it leaves the pitcher's hand until it is struck by the bat, because the only force acting upon it in the direction of motion is resistance, which is relatively small. A large momentum is given to a golf ball on a drive. Because the golf ball is in the air for a longer time than the baseball, much of its momentum gradually is transferred to the molecules in the air and apparently lost. However, the momentum gained by the molecules in the air is equal to the momentum lost by the ball. In this case, the isolated system must include both the golf ball and the surrounding air. Conservation of momentum applied to all isolated systems, whether the systems be colliding atoms, billiard balls, automobiles, or galaxies of stars.

FIG. 5

Consider an isolated system of boat and boater. The boater steps from the boat to a dock, as illustrated in Figure 5.

As the boater steps forward from the boat, a force is applied to the boat, as well as to the boater. The boater pushes back on the boat, but the boat pushes forward on the boater with an equal force. These forces cause the boater of mass m_1 to move forward with velocity v_1 at the same time that the boat of mass m_2 moves backward with velocity v_2. Because momentum is conserved, the product m_1v_1 (the boater's momentum) is equal to the product m_2 (the momentum of the boat): the initial momentum of boater and boat was zero before the boater stepped from the boat to the dock. After the boater steps from the boat, the total momentum must still be zero, since momentum is conserved. But the boater's momentum when stepping from the boat is $m_1\vec{v}_1$ while the boat's momentum is $m_2\vec{v}_2$. The sum of $m_1\vec{v}_1 + m_2\vec{v}_2$ must remain zero. Thus, $m_1\vec{v}_1 = -m_2\vec{v}_2$. The minus sign indicates that boater and boat move away from each other in opposite directions, each with the same numerical value of momentum. The total momentum of boater and boat in this system is zero before, during, and after the boater steps from boat to dock; and momentum is conserved.

An important and interesting observation to be made from the above example is that a boat with a much larger mass will "recoil" with a lower velocity. Thus, the velocity of the boat backward will be much greater for a lightweight canoe than for a heavy runabout. To step from a heavy boat is much easier than from a lightweight boat. Conservation of momentum in this example can be summarized by Equation 3.

$$m_1\vec{v}_1 = -m_2\vec{v}_2 \tag{3}$$

where

m_1 = mass of boater.
m_2 = mass of boat.
v_1 = velocity of boater.
v_2 = velocity of boat.

An important application of the conservation of momentum occurs in the firing of a rifle. When the rifle fires, the expanding gases push upon both the bullet and the rifle equally. The force produces a much larger velocity upon the small bullet than upon the massive rifle. As a result, the bullet leaves the barrel of the rifle with a very large velocity. The massive rifle, however, recoils with a much lower velocity. A relation similar to Equation 3 also applies to this case, since momentum is conserved. Equation 4 summarizes the results, and Example C shows how recoil velocity for a rifle can be calculated from Equation 4.

$$m_2\vec{v}_2 = -m_1\vec{v}_1 \tag{4}$$

where

m_1 = mass of bullet.
m_2 = mass of rifle.
v_1 = velocity of bullet.
v_2 = velocity (recoil) of rifle.

EXAMPLE C: CONSERVATION OF MOMENTUM AND RIFLE RECOIL VELOCITY

Given: A bullet of mass 5 g is fired from a rifle whose mass is 6 kg. The velocity of the bullet is 675 m/s when it leaves the rifle barrel.

Find: The recoil velocity of the rifle.

Solution: Both rifle and bullet considered separately — but as part of the same isolated system — are at rest before firing. Thus, the initial momentum of bullet and rifle is zero. Since momentum is conserved, the final momentum of bullet and rifle also is zero. Final momentum of the system (bullet and rifle) can be written as follows:

$$m_2v_{2\,\text{rifle}} + m_1v_{1\,\text{bullet}} = 0$$

or

$$m_2v_{2\,\text{rifle}} = -m_1v_{1\,\text{bullet}}$$

The negative sign is an important reminder that the recoil velocity of the rifle is in a direction opposite to the velocity of the bullet. The magnitudes of $m_2v_{2\,\text{rifle}}$ and $m_2v_{2\,\text{bullet}}$ are equal, so we can write:

$$|m_2v_2| = |m_1v_1|$$

The magnitude of the recoil velocity of the rifle is given by v_2, which is —

$$v_2 = \left|\frac{m_1v_1}{m_2}\right|$$

Substitution of the numerical values yields —

$$v_2 = \frac{5\text{ g} \times 675\ \frac{\text{m}}{\text{s}}}{6000\text{ g}}$$

$$v_2 = 0.56\text{ m/s}$$

The rifle kicks back with a recoil velocity of 0.56 m/s.

Another common application of the law of conservation of momentum involves the collision of two or more objects. Each object may have momentum before the collision, and each may have momentum after the collision. When Equations 3 or 4 are generalized to account for the collision of two or more objects, Equation 5 is obtained.

$$(\vec{P}_1 + \vec{P}_2 + \vec{P}_3 \ldots) = (\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots) \qquad (5)$$

where

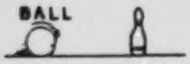
$\vec{P}$ = momentum of object(s) before collision.

$\vec{p}$ = momentum of object(s) after collision.

Equation 5 is used to solve a problem in Example D, which considers a bowling ball colliding with a stationary pin in such a manner that all motion takes place in a straight line.

EXAMPLE D: CONSERVATION OF MOMENTUM COLLISION WITH A STATIONARY OBJECT

Given: A 7-kg bowling ball moving with a velocity of 2 m/s collides head on with a bowling pin. Immediately after the collision, the 1.0-kg pin moves with a velocity of 6 m/s.



Velocities are in the same line (co-linear).

Find: Velocity of the ball immediately after the collision.

Solution: Equation 5 applies, since momentum must be conserved:

$$(P_{ball} + P_{pin}) = (p^1_{ball} + p^1_{pin})$$

Before After

P_{ball} = mass ball x velocity before collision.
= (7 kg)(2 m/s) = 14 kg-m/s.

P_{pin} = 0, since the pin has zero velocity before the collision.

p^1_{ball} = (7 kg)(unknown velocity, v_2).

p^1_{pin} = (1.0 kg)(6 m/s) = 6 kg-m/s.

Thus, use of the equation above yields —

$$(14 \text{ kg-m/s} + 0) = [(7 \text{ kg})(v_2) + 6 \text{ kg-m/s}]$$

Before After

Solve for v_2 —

$$(7 \text{ kg})(v_2) = 8 \text{ kg-m/s}$$

$$v_2 = 1.1 \text{ m/s}$$

The bowling ball thus continues straight ahead after the collision, with a speed of 1.1 m/s. Note: Friction of the ball against the floor and the spin on the ball may alter this result slightly. Friction represents an external force; therefore, this example is not a totally isolated system.

Momentum is conserved in all collisions. When a certain type of collision occurs, in which all the kinetic energy possessed by the objects remains as kinetic energy after the collision, it is called an "elastic collision." When some, or all, the kinetic energy is changed to other forms of energy during the collision, an "inelastic collision" results. (Kinetic energy can be changed to heat energy due to friction or can be used in changing the shapes of the objects on collision.) Of course, the total amount of all forms of energy is conserved in both types of collisions. Perfectly elastic collisions are not common and only occur in collisions between atoms and subatomic particles. Inelastic collisions are very common in all "real world" situations.

ANGULAR MOMENTUM

Angular momentum is very similar to linear momentum. In practice, it often refers to objects rotating about an axis. A massive flywheel rotating at the end of an engine crankshaft has large angular momentum. It is used to smooth the rotation of an engine between the firing of the pistons. Because of its large angular momentum, it tends to keep rotating evenly between firings, avoiding the otherwise jerky motion which would accompany the spaced firings of the pistons.

The mathematical relationship defining angular momentum is similar to the defining equation for linear momentum. Compare Equation 6 with Equation 1.

$$\vec{L} = I\vec{\omega} \qquad (6)$$

where

$\vec{L}$ = angular momentum.
I = moment of inertia.
ω = angular velocity.

In this equation, ω is the angular velocity, defined in Concept Module 3-0, "Rate." Angular velocity is the rate of angular rotation and in this equation must be measured in units of radians/s. The moment of inertia I is a quantity similar to mass in a translational mechanical system. The location of the axis of rotation of the object greatly affects the moment of inertia. If a small particle of mass, such as a grain of sand, is spinning about an axis a distance r from the grain of sand, the moment of inertia of the sand is equal to mr^2, where m is the mass of the sand particle. For large objects, the moment of inertia varies according to the shape of the objects and the location of the axis rotation. A few examples for moment of inertia of regular bodes are given in Figure 6.

Units for angular momentum can be determined by substitution into Equation 6:

$$L = I\omega \rightarrow (\text{kg-m}^2)(\frac{\text{rad}}{\text{s}}) \rightarrow \frac{\text{kg-m}^2}{\text{s}}$$

Thus, in the mk/s system, angular momentum has units of $\text{kg-m}^2/\text{s}$. In the cg/s system —

$$L = I\omega \rightarrow (\text{g-cm}^2)(\frac{\text{rad}}{\text{s}}) \rightarrow \frac{\text{g-cm}^2}{\text{s}}$$

Note: As discussed in Module 3-0, the radian is a "dimensionless unit" and can be omitted in expressions of units.

Example E indicates how angular momentum is calculated for a flywheel.

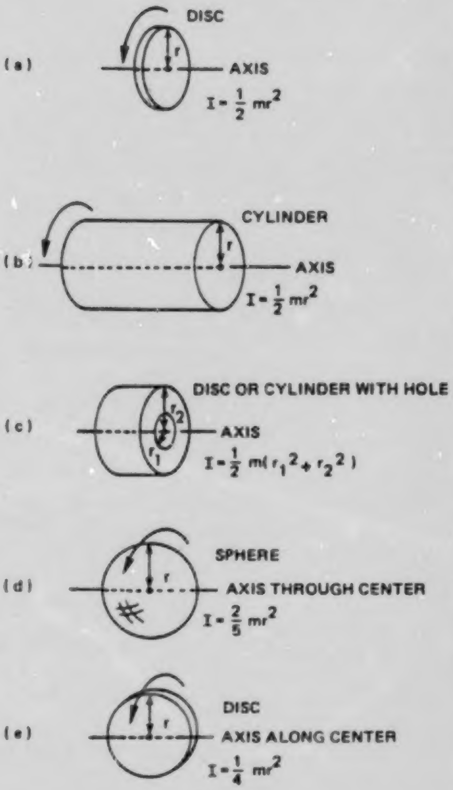
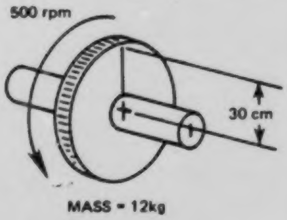

DISC
(a)
r
AXIS
$I = \frac{1}{2} mr^2$
CYLINDER
(b)
r
AXIS
$I = \frac{1}{2} mr^2$
DISC OR CYLINDER WITH HOLE
(c)
r_2
r_1
AXIS
$I = \frac{1}{2} m(r_1^2 + r_2^2)$
SPHERE
(d)
r
AXIS THROUGH CENTER
$I = \frac{2}{5} mr^2$
DISC
(e)
r
AXIS ALONG CENTER
$I = \frac{1}{4} mr^2$

FIG. 6

EXAMPLE E: ANGULAR MOMENTUM OF A FLYWHEEL


500 rpm
30 cm
MASS = 12kg

Given: A 30-cm (radius) flywheel with a mass of 12 kg is rotating about an axis through its center at 500 r/min (rpm).

Find: Its angular momentum.

Solution: $L = I\omega$

$I = 1/2\ mr^2$, since a flywheel is a disk rotating as shown in Figure 7a.

$$\omega = 500\ \frac{\text{r}}{\text{min}} \times \frac{1\ \text{min}}{60\ \text{s}} \times \frac{2\pi\ \text{rad}}{\text{r}} = 52.4\ \frac{\text{rad}}{\text{s}}$$

$$L = 1/2\ (12\ kg)(0.3\ m)^2(52.4\ \frac{rad}{s})$$

$$L = 28.3\ \frac{kg\text{-}m^2}{s}$$

Recall that linear momentum can be changed when a force is applied for a given time (impulse). Similarly, angular momentum can be changed when a torque is applied for a given time. Angular impulse is a torque (forcelike quantity) applied for a given time, as summarized in Equation 7.

$$\Delta L = \tau \Delta t \qquad (7)$$

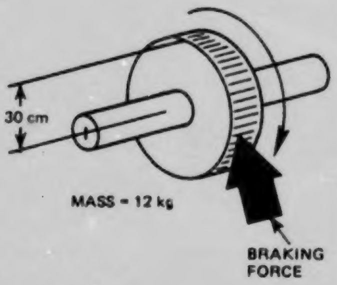
where
ΔL = change in angular momentum (angular impulse).
τ = torque.
Δt = elapsed time.

Relating this to the flywheel on the engine crankshaft, each time a piston fires, an angular impulse — a torque acting for a short time — acts on the engine shaft to increase its angular momentum. Since the moment of inertia of the crank shaft remains fixed, angular velocity is the quantity that increases. Angular velocity is transformed by the differential to increase the linear speed of the car. In Example F, the amount of angular impulse produced by a certain braking torque when applied to a rotating flywheel is examined.

EXAMPLE F: ANGULAR IMPULSE AND CHANGE IN ANGULAR MOMENTUM

Given: A braking torque of 5 N•m is applied to the flywheel in Example E for 2.5 seconds. Initial angular momentum, as worked out in Example E, is 28.3 kg•m^2/s.

Find: (a) The angular impulse produced and (b) the angular momentum of the flywheel after the braking action.



Solution: (a) $\Delta L = \tau \Delta t$

$= (5\ \text{N}\cdot\text{m})(2.5\ \text{s})$

$\Delta L = 12.5\ \text{N}\cdot\text{m/s}$

The units for angular impulse appear to cause trouble at first, because they are not the same as those for angular momentum. A short dimensional analysis is necessary to verify that these units are equivalent.

Substitution:

$$12.5\ \text{Newton}\ (\text{m/s}) = 12.5\ \left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right)(\text{m/s})$$

$$= 12.5\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

(b) Thus, the change in angular momentum of the flywheel is 12.5 $\text{kg}\cdot\text{m}^2/\text{s}$. The angular momentum of the flywheel after the braking action then is this:

original value - change = final value

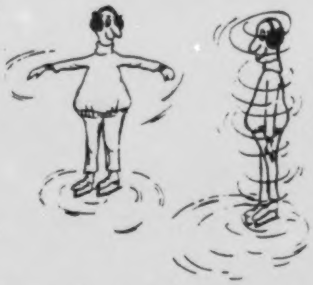
$$28.3\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}} - 12.5\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}} = 15.8\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

The law of the conservation of angular momentum is as important as the laws of conservation of linear momentum and the law of the conservation of energy. It simply states that "the angular momentum of a system remains constant, provided no external torques are applied"; that is, the system is an "isolated" one.

A complete discussion of the mathematical and vectorial analysis of angular momentum is beyond the objectives of this module; however, the results of the conservation of angular momentum in common applications are relevant.

An observation that can be made from Equation 6 (i.e., $L = I\omega$) is that several "hidden variables" are present. Both mass and radius of rotation are included in the moment of inertia term I. These facts are explained further in Examples G and H.

EXAMPLE G: ANGULAR MOMENTUM AND ICE SKATERS

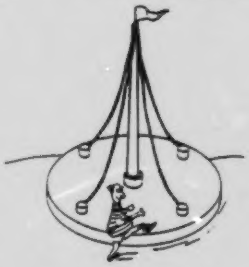


Given: Skaters practicing pirouettes extend their arms to slow down, and hold their arms tight to their body to increase their angular speed.

Find: The reason why.

Solution: A skater rotating must conserve angular momentum, like all other isolated systems. Thus, angular momentum while arms are extended must equal the angular momentum while arms are tucked in. When arms are extended, the effective radius of rotation of the skater becomes larger; thus, the angular velocity must be less for angular momentum to be conserved. In other words, the product $I\omega$ must remain constant; as I increases with a larger radius, ω must decrease. The same principle is used by platform divers and ballet dancers.

EXAMPLE H: MERRY-GO-ROUNDS AND ANGULAR MOMENTUM



Given: A merry-go-round turning with no one pushing will slow down when a heavy person gets on.

Find: The reason why.

Solution: The angular momentum of the merry-go-round before and after the additional person gets on must remain the same. When the heavy person gets on the merry-go-round, the mass of the system increases. This increased mass causes the value for I to increase in the equation $I\omega$; thus, ω must decrease to keep the product unchanged. The product must remain unchanged if angular momentum is to be conserved.

FLUIDAL SYSTEMS

When a stream of fluid hits a dish-shaped blade in the manner shown in Figure 7, the direction of the fluid is reversed.

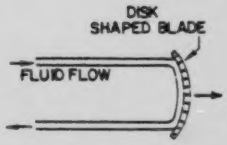


FIG. 7

The momentum of the moving fluid is changed, because its direction has been reversed. This change of momentum means that an impulse has been produced. The amount of force produced by this impulse can be deduced from Equation 2 and is given by Equation 8.

$$F = 2\left(\frac{m}{t}\right)v \qquad (8)$$

where
F = force on blade.
m = mass of fluid.
t = time.
v = velocity of fluid.

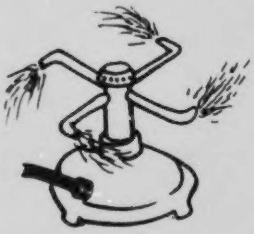


FIG. 8

This equation is the principle behind devices known as reaction turbines. The most common example of a reaction turbine is a common lawn sprinkler (Figure 8). The reaction to the change in momentum of the water as its direction is changed causes the arms to spin.

Any hose or pipe that forces a change in direction of a fluid causes a momentum change in the fluid and so must be strong enough to absorb the reaction to this momentum change. The reaction can be seen as an effort by the system to conserve momentum.

SUMMARY

The law of conservation of momentum is itself a unifying concept, because it is applied to any moving object or systems of objects free of external forces affecting the motion. This concept can include objects of all kinds, including moving fluids, moving electrons, or billiard balls. Momentum is used most often in connection with mechanical systems. The concept of momentum also is useful in connection with fluidal systems. The concept of momentum, at the level studied, is not found often to have practical application in the studies of heat or electricity.

LINEAR MOMENTUM

MATERIALS

Air track or air table and accessories (optional).
Two dry ice pucks (Cenco #75865-000 or equivalent).
Dry ice.
Two meter sticks.
Two stopwatches.

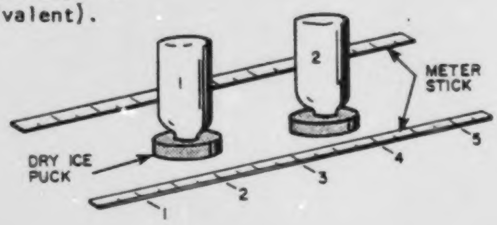


FIG. 9

PROCEDURE

If an air track or air table is available, use it to perform the momentum experiments suggested by the manufacturer or the equipment (optional).

Verify the conservation of linear momentum.

1. Position two identical dry-ice pucks on a smooth surface and between two meter sticks (Figure 9). The only purpose of the meter sticks is to act as tracks to assure linear motion of the pucks.

2. With puck #2 stationary and positioned approximately in center of the track, give puck #1 a slight push. Using a stopwatch, measure time required for puck #1 to travel distance between two points which you determine. From known distance and measured time, the average speed of puck can be calculated by the formula $v = d/t$. Record results in Data Table.

3. As puck #1 collides with puck #2, its momentum is transferred. But since the mass of puck #1 and puck #2 are equal, their speed is a direct measurement of their momentums. To verify the conservation of linear momentum, we must, therefore, show that the speed of puck #2 after the collision is the same as the speed of Puck #1 before the collision. Mathematically, this argument is as follows:

$$P_1 + P_2 = P_1{}^1 + P_2{}^1$$

$$mv_1 + mv_2 = mv_1{}^1 + mv_2{}^1$$

Since mass is equal for each puck, the following is true:

$$v_1 + v_2 = v_1^1 + v_2^1$$

4. Calculate speed of puck #2 after collision in same way that speed of puck #1 was calculated earlier.

5. Show that data verifies the conservation of momentum by substituting values into speed equation developed in Step 3.

DATA TABLE

Before Collision

Distance between marks 1 and 2 __________

Time for puck #1 to travel between marks 1 and 2 __________

Average speed of puck #1 before collision v_1 __________

Speed of puck #2 before collision $v_2 = 0$ __________

After Collision

Distance between marks 3 and 4 __________

Time for puck #2 to travel between 3 and 4 __________

Average speed of puck #2 after collision v_2^1 __________

Speed of puck #1 after collision v_1^1 __________

ANGULAR MOMENTUM

MATERIALS

One rotating platform (or stool) (Cenco #74790-000 or equivalent).
Two 2 kg masses.

PROCEDURE

Stand on a rotating platform (or sit on a rotating stool) with a 2 kg mass or more in each hand. Have a laboratory partner or instructor start the platform rotating. Change the position of your hands from a tucked-in position at your sides to a fully extended position, and observe the change in angular velocity.

How do your observations compare with the discussion of angular momentum presented in this module?

ELASTIC AND INELASTIC COLLISIONS

MATERIALS

One steel base, 2" x 2" x 1/2" or larger.
Equal-sized spheres (marble-size) of different materials (glass, steel, copper, lead, putty).

PROCEDURE

Obtain a relatively large, flat piece of steel. From identical heights, drop equal-sized spheres made of glass, steel, copper, lead, and putty onto the steel base. Measure the rebounding height of each sphere. On an arbitrary scale of 0 to 10, rate each collision, with 0 being an almost perfect inelastic collision (kinetic energy not being conserved) and 10 being an almost perfect elastic collision (kinetic energy completely conserved).

SELF CHECK

1. Define the following terms; state both the mk/s and the cg/s units for each; and give the equation for each.

   Linear momentum.
   Moment of inertia.
   Angular velocity.
   Angular momentum.
   Impulse.
   Angular impulse.

2. What is the momentum of a 250-lb object moving with a velocity of 5 ft/s? (Hint: Be sure to convert 250-pound weight to mass in slugs.)

3. If a rifle were lighter than the bullet it fired, which would be safer — the rifleperson or the target? Explain your answer in terms of the law of conservation of momentum.

4. A proton (mass = 1.67×10^{-27} kg) moves with a velocity of 5×10^6 m/s. Upon colliding with a stationary particle of unknown mass, the proton rebounds along its original path with a velocity of 3×10^6 m/s. The collision sends the unknown particle forward with a velocity of 1.5×10^6 m/s. What is the mass of the unknown particle?

5. A projectile with a mass of 0.5 kg and a velocity of 700 m/s collides with a 20-kg mass initially at rest. The projectile imbeds itself within the mass. What is the velocity of the 20-kg mass and projectile after the collision? (Remember this: Momentum before equals momentum after!)

6. The radius of a solid shaft is 10 cm, and its mass is 5 kg. It is rotating with an angular velocity of 1800 r/min (rpm) about an axis of rotation through its center, as is shown in Figure 6b.

   a. What is the angular momentum of the shaft?
   b. What angular impulse is needed to increase the angular velocity to 2400 r/min?
   c. What constant torque is needed to increase the angular velocity from 1800 r/min to 2400 r/min in 30 seconds?

7. An Olympic high diver changes his speed of rotation while in flight by tucking himself into a small ball or by straightening himself out. Explain why this is possible in light of the conservation of angular momentum.

8. Use Equation 8 to calculate the force produced on one blade of a reaction turbine when 5 kg per second of water strikes the blade with a velocity of 7 m/s.

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RESISTANCE

CHAPTER 5-0

Motion of a moving system gradually dies out unless a forcelike quantity is present to maintain it. A small child in a swing must be pushed continually to be kept moving; otherwise, oscillations of the swing grow smaller and smaller, until all motion ceases. A phonograph turntable will slow down and stop when its electrical power is turned off. A little reflection will convince one that when a moving system is not assisted by some forcelike quantity, the motion gradually decreases until it stops.

Some systems maintain motion without assistance for a longer time than others; but in all cases, motion ultimately ceases if no forcelike quantities are present to maintain it. This dying out of motion is the result of resistance within the system. Resistance appears in different forms in different energy systems, but in all cases creates forcelike quantities that oppose motion within the system.

This module discusses (1) the phenomenon referred to in a general way as "resistance" and (2) the application of the concept of resistance to mechanical, fluidal, electrical, and thermal systems. The discussion includes the similarities of resistance in all energy systems. These similarities make resistance a unifying concept. The discussion also includes the differences in resistance characteristics from one system to another.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define resistance in a general way and state the final form of the energy expended when a forcelike quantity does work to overcome resistance in any energy system.

- Calculate the magnitudes of starting and sliding frictional forces, given the mass or weight of the object, the coefficients of friction, and the angle of incline.

- Given two of the following quantities in fluidal, electrical, and thermal systems, determine the third:

    Forcelike quantity.
    Rate.
    Resistance.

- Describe the differences in laminar and turbulent flow.

- State the factors contributing to fluid resistance in pipes.

- State the factors contributing to electrical resistance in wires.

- State the factors contributing to the thermal resistance of objects.

- Describe with the use of graphs the definition of resistance as the ratio of forcelike quantity to rate in fluidal, electrical, and thermal systems. Include the units of forcelike quantity, rate, and resistance for each system.

- State the fundamental difference between sliding friction and resistance as it applies to fluidal, electrical, and thermal systems.

DISCUSSION

Resistance is an opposition to motion. It acts to oppose the forcelike quantities in all energy systems, thus causing a reduction of rates. If no forcelike quantities are present to counteract the effects of resistance, motion in the system will die out gradually and eventually will cease altogether. Although some systems maintain motion without assistance for a longer time than others, motion ultimately ceases in all instances.

If a moving system is to be maintained in motion at a constant rate, a forcelike quantity must act continuously upon the system and, in acting upon a moving system, must do work. The work done against friction in all energy systems is converted to heat energy. This process is taken advantage of in systems in which the conversion of other forms of energy to heat energy is desirable. In other cases, the energy converted to heat is wasted and cannot be recovered. Often, this heat energy causes thermal problems in a system, and steps must be taken to remove it. A stream of water used to cool the edge of a glass-cutting wheel is an example.

The following are examples of the effects of resistance in the four energy systems discussed in this module:

- In mechanical systems, resistance appears as friction between any two surfaces in contact. The effect of friction is to slow the motion of moving objects and to prevent the motion of stationary objects. Lubricants are used to reduce frictional forces and, thus, to reduce the force necessary to maintain motion and the amount of energy converted to heat.

- In fluidal systems, resistance appears as drag (fluid friction) forces on solid objects moving through fluids and as fluid resistance that limits fluid-flow rates through pipes. Much of the energy used by an automobile appears as work done against the fluid friction of the air.

- In electrical systems, electrical resistance opposes current in conductors. In electrical power distribution systems, this resistance must be minimized to reduce energy losses. In heating devices, such as toasters and electric heaters, resistance is employed to convert electrical energy into heat energy.

- In thermal systems, resistance appears as the opposition to heat flow. Materials, such as metals, that transmit heat readily have low thermal resistance. Thermal insulators are materials having high resistances to heat flow. In this system, resistance acts to reduce the rate of motion only. There is no energy available to be converted into heat, since only heat energy is present in thermal systems.

All these points bring to mind a fascinating question, not of vital concern at this stage of study; but just pause for a moment to ask it. "Is a completely frictionless system possible?" If such a system could be developed or found, no energy waste would occur due to resistance effects; in other words, the system would be 100% efficient. In fact, sometimes such an ideal situation can be approached very closely. For example, a rocket ship moving through the near-vacuum of outer space encounters very little resistance to its motion and coasts for great distances without additional energy input. In electrical systems, an electrical current can be maintained for a very long time without additional energy input in the "resistanceless" circuit made of a superconductor. The fact remains, however, that a perfectly frictionless system is an ideal.

In summary, a moving system involves some degree of resistance to motion. To maintain motion, additional energy input to the system is required. Where the heat produced by this additional energy is not desirable, it represents a waste of energy; and an effort is made to reduce resistance of the system. There are cases, however, when the heat produced is taken advantage of and put to some useful purpose, as for example, with industrial heat exchangers.

The remainder of this module discusses in greater detail the types of resistance introduced above.

MECHANICAL SYSTEMS - DRY FRICTION

Bodies in contact, and in relative motion with respect to one another — that is, rubbing against each other — produce a resistance to the motion, usually called "dry friction." This term is applied to distinguish this type of friction from the friction that a body experiences as it moves through a fluid. Such fluid friction often is called "viscosity," which will be considered later.

A simple and effective way for the student to understand this discussion is to think of a block of wood being dragged along the top surface of a table. A string is tied to the side of the block, and the block is being pulled in a horizontal direction (Figure 1). If a spring scale is introduced between the hand and the string, the scale will register the force required to drag the block along the table. The force that must be applied to do nothing more than slide the block along at constant speed is a measure of the frictional force resisting the motion. This frictional force originates at the point where the block surface and table surface are in actual contact. Microscopic inspection of this region shows that the surfaces are far

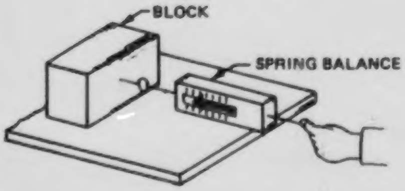


FIG. 1

from smooth: both surfaces have small and irregular "bumps." When stationary, many of the small bumps, or protrusions, of the block surface "lock" into corresponding holes in the table surface, and vice versa (Figure 2).

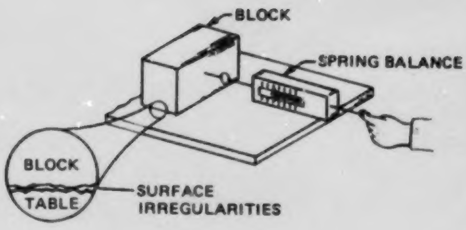


FIG. 2

A certain amount of force is required to overcome the physical barriers (resistance) to motion provided by these tiny irregularities. Even surfaces that appear to be very smooth are, in fact, irregular. No surface can be perfectly smooth, because matter is ultimately granular in nature. At any rate, the smoother the surfaces, the less the frictional drag.

Dry friction, therefore, depends upon the types of surfaces involved. Wood dragged along a smooth tile floor involves less friction than wood dragged against a wooden floor, and so on. How the lubrication of surfaces reduces frictional force should be obvious now. If the table top is covered with a film of liquid or oil, for example, the film can act to hold the block and table surfaces apart enough to reduce collisions of their irregular surface bumps (Figure 3).

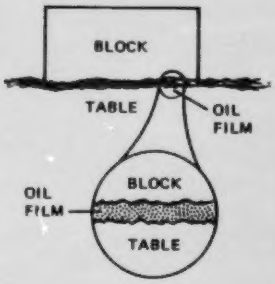


FIG. 3

Apart from the types of surfaces involved, what other factors might influence frictional force? Obviously, weight (W) of the block makes a difference. A heavy crate is more difficult to drag than a light crate of the same size and shape. More precisely, the frictional force really depends upon the force pushing the two surfaces together. This force usually is called the "normal force" (N), because its direction is perpendicular, or "normal," to the surface area. This bit of precision is important, because weight W and normal force N are not always identical. Example A shows cases in which weight and normal force are the same and in which they are different. Example A also shows the importance of treating forces as vectors and of being able to resolve such vectors into components.

EXAMPLE A: NORMAL FORCE PROBLEM

Given: Two crates weigh 100 pounds each. One rests on a horizontal surface and the other on a surface 30° from horizontal.

Find: The normal force on each crate.

Solution: (a) In this case, weight of the crate acts perpendicularly to the horizontal floor. Thus, weight and normal force are identical. W = 100 pounds. N = 100 pounds.

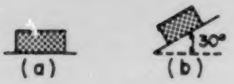
(b) In this case, weight continues to act vertically downward. Its component perpendicular to the surface is less than the weight and can be found from the vector triangle.

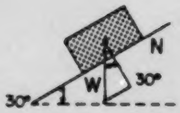
$N = W \cos 30°$

$= (100)(\cos 30°)$

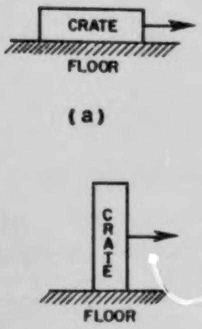
$= (100)(0.866)$

$N = 86.6$ pounds





What factors other than the types of surfaces and the normal force pressing them together are involved in determining the frictional force? Surprisingly, nothing else matters much. For example, consider whether or not the amount of surface area itself makes a significant difference. Consider a crate being dragged along the floor, first on its large side and then on its small side (Figure 4). One side (a) has a large surface area, the other (b) a small surface area in contact with the floor. Of course, weight of the crate does not change. There is no significant difference in the frictional forces acting. This may seem rather remarkable at first, because our understanding of the source of friction would lead us to believe that the larger surface area in contact with the floor would have more friction. There are more surface irregularities to become locked together. But, what we have overlooked is that, in this case, there is less weight directly above the irregularities pressing them into the floor. Apparently, these two effects cancel each other. Thus, dry friction is independent of the contact surface area.



(a)

(b)

FIG. 4

Consider whether the speed with which the block is dragged along makes a difference. Again, an experiment would show that speed is not important — unless speeds are reached at which air friction is no longer negligible. Therefore, dry friction is independent of velocity.

The conclusion can be made that dry-frictional forces depend upon two factors:

(1) Nature of the sliding surfaces.
(2) Normal force pressing them together.

Further experiments show that, in fact, frictional force is directly proportional to the normal force; that is, if the normal force is doubled, frictional force also is doubled. All this information is summarized in Equation 1.

$$f = \mu N \qquad (1)$$

where

f = frictional force.
N = normal force.
μ = "coefficient of friction."

The coefficient of friction μ depends upon the nature of the two surfaces involved. It is a pure number without units, being simply the ratio of two forces, f/N. Some approximate values for the coefficient of friction for common materials are given in Table 1. Example B illustrates the usefulness of Equation 1.

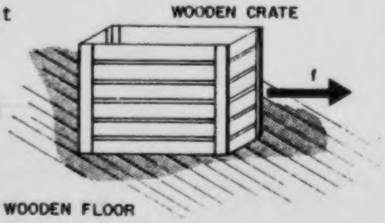
TABLE 1: COEFFICIENTS OF SLIDING FRICTION (μ)

Coefficients of Sliding Friction (μ) (Values are approximate.)	
Wood on wood	0.3
Wood on stone	0.4
Metal on metal	0.15
Metal on wood	0.5
Leather on wood	0.4
Rubber on dry concrete	0.7
Rubber on wet concrete	0.5

EXAMPLE B: DRY SLIDING FRICTION

Given: The 100-pound crate of Example A has a coefficient of friction between crate and floor of 0.25.

Find: What force must be applied to overcome friction and to keep the crate sliding at constant speed in each case, a and b.



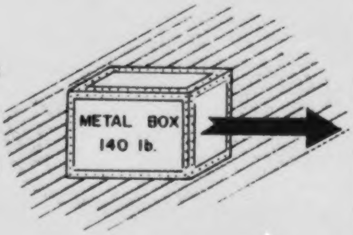
Solution: (a) $N = 100$ pounds, $\mu = 0.25$
$f = \mu N = (0.25)(100) = 25.0$ pounds

(b) $N = 86.6$ pounds, $\mu = 0.25$
$f = \mu N = (0.25)(86.6) = 21.65$ pounds

There is a small complication in this simple theory. A larger force is required to start a box sliding than to keep it sliding. In fact, more effort is required to unlock the two surfaces when they are stationary than to keep them sliding when motion has begun, which means that the differences between coefficients of "starting friction" and coefficients of "sliding friction" must be identified. The first is called a static friction coefficient μ_s and the second a kinetic friction coefficient μ_k. For the same two surfaces, μ_s will be slightly greater than μ_k. Consequences of the difference in μ_s and μ_k are illustrated in Example C.

EXAMPLE C: STATIC AND KINETIC DRY FRICTION

Given: A metal box weighing 140 pounds rests on a horizontal wooden floor. The coefficient of static friction for these surfaces is 0.55, and the coefficient of kinetic friction is 0.50. A pull rope is attached.



Find: What happens as pull on the rope is increased.

Solution: Static frictional force $f_s = \mu_s N = (0.55)(140) = 77$ pounds

Kinetic frictional force $f_k = \mu_k N = (0.50)(140) = 70$ pounds

Therefore, no motion of the box occurs until pull on the rope reaches 77 lbs. At this point, the box begins to slide. Since pull required to keep the box sliding is only 70 lbs, the box will begin to increase in speed, or accelerate, unless the pull is reduced to 70 lbs. If the pull is reduced below 70 lbs, however, the box gradually will come to a stop.

Still another type of friction occurs when one object is rolling along the surface of another. The same general equation can be used, but proper value of the coefficient of "rolling friction" to be used must be determined.

Sliding friction, remember, is not dependent upon the velocity of the surfaces involved. Figure 5 is a graph of frictional force versus velocity. Static friction is the maximum possible value of frictional force that can exist between stationary surfaces for a given normal force. If a force applied parallel to the surface exceeds the magnitude of the static friction, motion results. Once motion has begun, the frictional force drops to the sliding friction value and remains there as long as the surfaces are in motion. Sliding friction and rolling friction are the only types of resistance that do not change as rate changes.

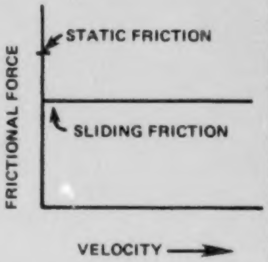


FIG. 5

FLUIDAL SYSTEMS

Friction or resistive forces also occur in fluids (liquids or gases). We distinguish two cases: (1) a solid body moving through a fluid and (2) the internal friction, or friction of a fluid with itself, usually described as the "viscosity" of the fluid.

RESISTANCE TO MOTION OF A SOLID BODY THROUGH A FLUID - FLUID FRICTION

A solid body that forces its way through a fluid, like a ship is forced through the ocean, must do the work of separating the fluid to allow passage. Thus, the moving object encounters resistive forces, or fluid friction, in its motion. A minimal amount of energy loss occurs when the water is simply separated and then comes together again after the passage, without the creation of eddies and other kinds of swirling motion of the fluid. The simpler motion of the fluid is accomplished by the streamlining of the shape of the object. Unlike the case of dry friction, the shape of the object now is all important. Much is learned about streamlining from the natural shapes of birds and of fish as they move effortlessly through air and water. Their shapes are compared with the present designs for automobiles and especially for jet airplanes. Turbulence (swirling) is more difficult to avoid at high velocities; therefore, streamlining is especially important in the design of vehicles that must move at relatively high speeds through air or water. In contrast, recall the cumbersome shapes of weather satellites. Although they move in orbit at tremendous speeds, they move through a near vacuum — that is, the total absence of air — so that streamlining is not important.

At higher speeds, more turbulence is created; therefore, more energy is dissipated because of fluid friction in heating the fluid. This additional energy is wasted. To operate an automobile running at high speeds costs much more than at low speeds, as evidenced by the poorer figure for miles per gallon of gasoline in operating an automobile against air friction at higher speeds. Or equivalently, recall the difference in effort required in walking through water and in running through it at the beach.

The sport of skydiving (Figure 6) illustrates the increase of fluid friction with velocity. A skydiver who falls a great distance before opening the parachute quickly reaches a steady velocity of fall — a "terminal velocity" of about 130 miles per hour. From the moment of jumping, the diver's speed increases because of weight (that is, the pull of the earth). At the same time, the velocity and the frictional force of air are increasing. When this upward force of air (drag) becomes equal to the downward force (caused by the diver's weight), these forces cancel each other; and the diver maintains a certain velocity. Before hitting the ground, pulling on the rip-cord releases the parachute. The parachute is anything but streamlined in shape, and it increases air drag to such an extent that the skydiver continues to fall at a much smaller terminal velocity of only a few feet per second.

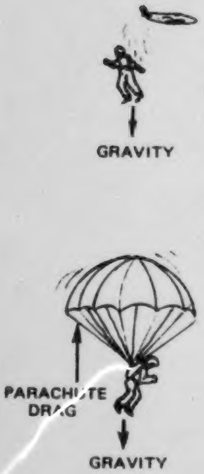


FIG. 6

Unlike dry friction, fluid friction depends upon velocity of the motion of a body through the fluid. If the body is well streamlined and if fluid moves past the body without turbulence, the fluid flow is called "streamline," or "laminar" flow. In this case, frictional forces are very nearly proportional to the velocity; that is, frictional force doubles when the velocity doubles. As velocity is increased further, however, turbulence eventually will be encountered; and then the frictional forces will increase more rapidly with velocity. For example, air drag on an airplane wing is proportional to the square of the velocity: frictional forces become four times as great when the speed doubles. At even higher speeds, the forces may vary as the cube or higher powers of the velocity. Velocity is an important factor, therefore, in the determination of fluid friction, but the exact equation depends very much upon conditions, and must be determined experimentally.

Fluid friction presents a way of thinking about the resistance of a system that it will be valid in other systems as well. The following paragraphs present an experiment and a graphical analysis of the results. The relationships examined will be applied to other energy systems later in this module. Although these experiments can be done in principle, some of the measurements would be difficult to make because of the choice of materials tested. The materials will be ignored, since the principles are more important.

Imagine taking data and plotting it in the following simple experiment, with the objective of determining the frictional force on a solid body moving through a specified liquid. A cylinder (Figure 7) is filled with a fluid, water. A metal sphere suspended on a string is raised at constant speed, encountering fluid frictional drag. The spring scale registers pull of the string P upward on the sphere. Pulling downward on the sphere are its weight W and the frictional drag f. When weight and measured force P are known, the frictional drag can be determined from f = P - W. (To be more exact, a buoyant force P also forces the sphere upward.) The buoyant force would equal the weight of the fluid that the small sphere displaces. Data is taken in this way for values of f corresponding to different velocities of rise v. The plotting of all these points gives a graph that looks something like that shown in Figure 8. Each point represents a corresponding pair of values of v and f. The set of points appears to determine a straight line drawn through the points.

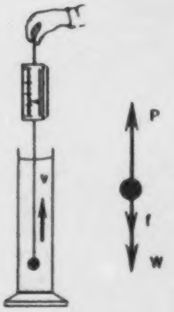


FIG. 7

Next, imagine that the experiment is repeated, but that the cylinder is filled with molasses, and then with alcohol. If all these graphs are plotted together, something like Figure 9 would result. Each straight line has a different inclination, or slope, the significance of which will be investigated.

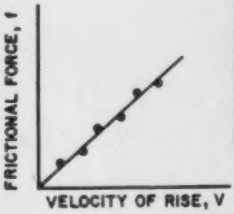


FIG. 8

At a given velocity of rise, like v_1, the graph shows that molasses presents the greatest frictional force, water the next largest, and alcohol the least. These values are shown on the graphs as f_m, f_w, and f_a, respectively. Therefore, the line with the greatest slope indicates a fluid that offers greater fluid friction. A quick glance at the curves then shows that molasses is more viscous than water and water more viscous than alcohol (as might be expected); therefore, when forces are plotted against velocities in this way, slopes of the graphs are a measure of resistance of the system. If velocities used in this experiment were large enough to create turbulence, the graphs would be curved rather than straight. When force is proportional to velocity, the graph is a striaght line; when force becomes proportional to the second or third powers of velocity, the curves deviate from straight lines.

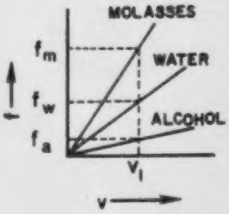
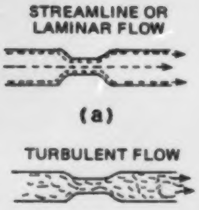


FIG. 9

FLUID RESISTANCE IN PIPES

When a fluid is flowing through a pipe, certain parts of the fluid are moving faster than other parts, so that internal fluid friction is involved. Fluids with high internal friction, like molasses, have a high viscosity; those like alcohol or gases have a much smaller viscosity. As with coefficients of dry friction, viscosity can be defined more precisely and measured, enabling introduction of a useful number called the "coefficient of viscosity."



(a)

(b)

FIG. 10

Energy losses due to the viscosity of fluids become important when fluids must be transported over great distances. Consider pipelines that carry petroleum, natural gas, and water across large sections of the country. The Alaskan pipeline is an excellent example. Energy losses due to fluid friction must be compensated for by extra pumps at regular intervals along the pipelines to maintain necessary pressure. The energy required to operate these pumps is essentially the energy lost by the fluid due to viscosity. These energy losses are greatest when flow is turbulent. Figure 10 shows a section of pipeline that includes a constriction. The dotted lines in Figure 10a indicate paths of fluid particles in the case of streamline or laminar flow, and then in the case of turbulent flow (10b). The portions of the fluid near the inner pipe wall move more slowly than the fluid near the center of the pipe, meaning that between different portions of the fluid there is relative motion, which gives rise to fluid friction. In streamline flow, these portions, or layers, of fluid simply slide past one another. In turbulent flow, complex motions that waste energy occur.

For occasions when fluid flow is without turbulence (streamline), the average resistance to fluid flow can be calculated with Equation 2.

$$R_F = \frac{\Delta p}{Q_V} \qquad (2)$$

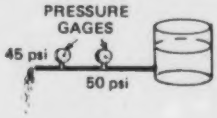
where

R_F = fluid resistance.
Δp = pressure difference across pipe.
Q_V = volume flow rate; velocity.

The resulting units for resistance will be pressure units (N/m^2 or lb/in^2) divided by flow rate units (gal/min or m^3/s). The use of this equation is illustrated in Example D.

EXAMPLE D: FLUID FLOW PROBLEM

Given: A pipe has an inlet pressure of 50 psi and an outlet pressure of 45 psi. Fluid is flowing in the pipe at the rate of 25 gal/min.



Find: Internal resistance of the pipe.

Solution:

$$R_F = \frac{\Delta p}{Q_V}$$

$$= \frac{50\ lb/in^2 - 45\ lb/in^2}{25\ gal/min}$$

$$= \frac{5\ \text{lb/in}^2}{25\ \text{gal/min}}$$

$$R_F = 0.2\ \frac{\text{lb/in}^2}{\text{gal/min}}$$

This answer means that the resistance of the pipe is 0.2 lb/in^2 for every gal/min velocity.

When fluid flows through a pipe, there is a frictional drag between it and the inner wall of the pipe and a viscous drag between successive layers of fluid from the wall inwards to the center, where flow velocity is greatest. To maintain the flow, pressure (forcelike quantity in the fluid system) on the input end must be greater than pressure on the output end.

Suppose our pipe is just a glass tube at the base of a cylinder of water (Figure 11). The pressure differential across the pipe, $p_1 - p_2$, is then just due to the head of water in the cylinder, and is recorded simply in units of inches of water head. If the valve at the end of the pipe is opened for a measured time interval and the outflow of water is collected and measured, volume rate of the flow in milliliters per minute or gallons per hour is determined. If collecting time is short enough, the reservoir level will not drop much, and the water is assumed to be collected at a constant pressure differential. Data taken in this way is plotted as pressure differential (forcelike quantity) against flow rate. Measurement of a different flow rate for each different height of water (for three different pipes of different diameter) will result in three separate graphs as before (Figure 12).

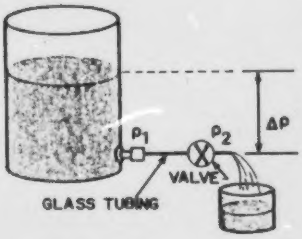


FIG. 11

The pipe represented by the line of greatest slope is the one that offers the greatest resistance. For a given flow rate, pipe No. 1 evidently requires the greatest pressure differential to overcome its own resistance to fluid flow. As before, when forcelike quantity is plotted against ratelike quantity, a graph is arrived at in which the slope is a measure of resistance offered by the system.

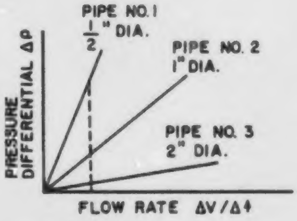


FIG. 12

The value of fluid resistance in a long, straight, smooth pipe depends upon the following three factors:

- Length of the pipe. Fluid resistance increases linearly with pipe length. If the length is doubled, the resistance doubles.

- Area of the pipe. The area of the pipe increases as the square of the diameter, and fluid resistance decreases as the square of the area. Thus, doubling of the pipe diameter increases the area by a factor of 4 (2^2) and decreases the resistance by a factor of 16 (4^2).

- Viscosity of the fluid. Fluid resistance increases with an increase in viscosity.

Roughness of the pipe inner walls, bend, and constrictions also add to fluid resistance.

ELECTRICAL SYSTEMS

An electric current in a wire is an orderly motion of electrons in the wire. For direct current, this motion is a steady progress of electrons from one end to the other. For alternating current, there is an orderly motion of electrons back and forth at the frequency (usually 60 cycles per second) of the electrical system. Of course, these electrons undergo collisions and random motions in the process. But when a current exists, the electrons experience a net drift through the material. In contrast, when no current exists in the wire, motion of the electrons is random; and the average effect upon their motions is simply no motion at all.

Because these electrons are "free" to wander through the bulk of the material without being captured by any one atom in particular, they contribute to both heat and electrical conduction. Good conductors of heat (metals) usually make good electrical conductors, and vice versa. Materials differ widely in their ability to conduct electricity — from copper, near the good-conductor end of the spectrum, to hard rubber, near the good-insulator end. Even liquids (salt water, for example) make rather good electrical conductors. In this case, individual electrons do not carry the current, but charged atoms or ions that do the job.

An experiment is now described that will enable electrical resistance to be presented in a manner analogous to the fluid resistance in pipes previously discussed. In Module 1-0, force-like quantity in the electrical system that produces charge transport, or electrical current, is described as a potential difference V, in volts. The ratelike quantity is the rate at which charge is transferred through the material, which is precisely what is meant by electrical current. The most common unit of current is the number of coulombs of charge per second, which are called amperes of current.

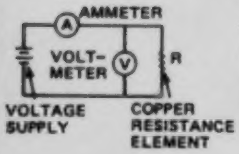


FIG. 13

Consider a copper element (a piece of copper of a particular size and shape) included in an electrical circuit (Figure 13) made up of a variable voltage source and two measuring instruments — a voltmeter to measure potential difference across the copper element, and an ammeter to measure charge flow rate or current through the element. Symbols used for a battery and for a resistor R represent the copper element.

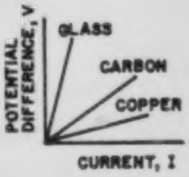


FIG. 14

Simply vary the potential difference V and read the corresponding value of current I off the ammeter. If this data is plotted as shown, a straight-line graph results for copper.

Data is taken for a sample of glass and for a sample of carbon, both cut to the same size and shape as the copper, so as to compare the results easily. By now, these results should be meaningful. Plotted in Figure 14 is the forcelike quantity V against the ratelike quantity I. The slope of these lines is again a measure of the resistance. Evidently, glass has more resistance to electrical conduction than does carbon. Copper turns out to be the best conductor of the three, as might be suspected.

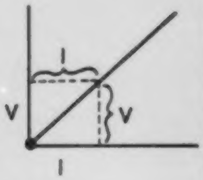


FIG. 15

The slope of these straight lines is found, as usual, by taking the interval ΔV between any two points on the line and dividing by the corresponding interval ΔI. One point of the interval is chosen as the origin, shown in Figure 15; then, slope = V/I. As discussed previously, the slope is a measure of resistance of the system. Here electrical resistance R is defined as the ratio of V/I, as in Equation 3.

$$R = \frac{V}{I} \tag{3}$$

where

R = resistance.
V = voltage.
I = current.

If V is in volts and I is in amperes, R is defined in terms of a unit called the "ohm." One ohm of resistance is the resistance of an element in which one ampere of current results when one volt of potential difference exists across it. Briefly, an ohm is one volt per ampere. The name "Ohm" was chosen in honor of the discoverer of this basic law, the German scientist Georg Simon Ohm. This law is referred to as "Ohm's law." Examples E and F illustrate use of Ohm's law to calculate resistance and current in electrical circuits.

EXAMPLE E: OHM'S LAW

Given: The circuit described above, with a current of 2 amperes when the voltage is 10 volts.

Find: Resistance of the resistive element.

Solution: The voltage, current, and resistance are related simply through Ohm's law, $V = IR$ or,

$$R = V/I = (10 \text{ volts})/(2 \text{ amps}) = 5 \text{ ohms}$$

EXAMPLE F: OHM'S LAW

Given: Voltage is reduced to 4 volts.

Find: Current through the same resistor.

Solution: Ohm's law is applied again. This time, current is the unknown, so the law solving for I is used thusly:

$$I = V/R = (4 \text{ volts})/(5 \text{ ohms}) = 0.8 \text{ amps}$$

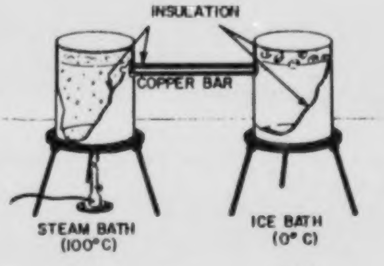
The resistance of electrical conductors depends upon these three quantities:

- Length. The resistance of a wire increases linearly with its length. If length is doubled, resistance doubles.
- Area. Resistance decreases proportionally to the area of the conductor. If the area is doubled, resistance is reduced by one-half. If the diameter is doubled, the area increases by a factor of 4 and reduces the resistance to one-fourth the initial value.
- Resistivity of the conductor. Resistivity is a characteristic of the material from which the conductor is constructed. Copper has a low resistivity, and glass has a high resistivity. Resistance increases as resistivity is increased.

THERMAL SYSTEMS

Resistance in a thermal system can be understood in an analogous way. In many ways, the flow of heat energy through a bar is like the flow of fluid through a pipe or of change through a wire. As explained earlier in Module 1-0, the forcelike quantity that causes the flow in this case is a temperature difference ΔT, and the ratelike quantity is the heat flow rate Q_H (for example, in calories per second). The experiment in this case is much like the fluid-flow experiment in the previous section.

In Figure 16, there is a copper bar that conducts heat from its left end, held at an elevated temperature, to its right end, held at a lower temperature. To begin with, the left end of the



(a) (b)

FIG. 16

bar is in thermal contact with a steam bath, and the right end is in thermal contact with an ice bath. The steam bath, ice bath, and copper bar are all well-insulated. At atomspheric pressure, a steam bath at one end, and an ice bath at the other end would amount to a temperature differential of 100 C°. Heat energy associated with greater atomic vibrations at the left end gradually is transferred to the right end, raising its temperature. After the bar is left in this condition for some time, it comes to equilibrium — "steady state" condition reached, in which the temperature at each point of the rod no longer is changing, from the hot end to the cold end. The rate at which heat energy is being conducted through the rod under these steady-state conditions is of interest. Ways of measuring this quantity will depend upon the ingenuity of the experimenter. For simplicity, assuming adequate insulation of all components, the amount of ice melted in a given amount of time is measured. Known is amount of heat required to melt each gram of ice (the "heat of fusion" of ice is 80 calories per gram); also known is how much heat has been transferred in the given time interval. Therefore, the heat flow rate Q_H can be calculated. All that is necessary now is to change temperature of the constant-temperature bath at the left (Figure 16a), giving a different value for ΔT, and then measure the corresponding quantity Q_H from the ice melted on the right end (Figure 16b). In this way, pairs of data points for ΔT and Q_H are collected and plotted for the copper bar.

If this process is repeated for other materials, such as a glass rod and a wooden rod, all the same length and cross section, a graph something like that shown in Figure 17 is produced. Interpretation proceeds as it did for the other systems. The fact that for a given flow rate, a larger temperature difference is needed for cork than for either glass or copper clearly means that cork offers the greatest resistance to heat flow. Of the three, copper offers the least resistance. Again, the slope of the lines is a measure of resistance offered by the thermal system. Of these three materials, cork is the best insulator, and copper is the best conductor.

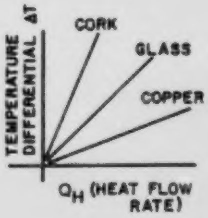


FIG. 17

Materials possessing a high resistance to heat flow are good insulators. Materials like cork, styrofoam, fiberglass, and even layers of air keep heat out (as in a refrigerator) or to keep heat in (as in homes). At other times, good conductors are desired to transfer heat. Metals make the best conductors, because they contain many free electrons that aid in transfer of heat energy.

Everyday items, such as metal radiators or the copper bottoms of tea kettles, are good examples.

Thermal resistance is defined by Equation 4.

$$R_T = \frac{\Delta T}{Q_H} \qquad (4)$$

where

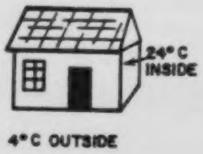
R_T = thermal resistance.
ΔT = temperature difference.
Q_H = heat flow rate.

The units of thermal resistance are temperature difference units (F° or C°) divided by heat flow rate units (Btu/hr or cal/g).

The use of this equation is illustrated in Example G.

EXAMPLE G: THERMAL RESISTANCE

Given: Temperature inside a house is 24°C; outside, the temperature is 4°C. A furnace supplies 2625 kcal to the room each hour to maintain the inside temperature of 24°C.



Find: Thermal resistance of the house.

Solution:

$$R_T = \frac{\Delta T}{Q_H}$$

$$= \frac{24°C - 4°C}{2625\ \text{kcal/h}}$$

$$= \frac{20°C}{2625\ \text{kcal/h}}$$

$$R_T = 0.0076°C(\text{kcal/h})$$

This value tells us that the house gains 0.0076°C over the 4°C outside temperature for every kcal/h of heat flow from the furnace.

The thermal resistance of a sheet of material depends upon the following three quantities:

- Thickness. The resistance to heat flow increases linearly with thickness. The doubling of thickness doubles resistance.
- Area. The thermal resistance decreases linearly with increasing area. When area is doubled, resistance is halved.
- Thermal conductivity. If conductivity is increased (when materials are changed), resistance decreases.

SUMMARY

As shown previously, in different physical systems, an important phenomenon shows up as a resistance to motion with a corresponding energy loss, called the "characteristic" of the system resistance.

In the case of sliding friction, resistance is independent of velocity (rate). In fluidal, electrical, and thermal systems, the resistance can be analyzed and measured graphically by the plotting of (Figure 18) the forcelike quantity (y-axis) against the ratelike quantity (x-axis) for the particular system. Resistance of the system then is the slope of the curve. Usually, the curve is a straight line of constant slope passing through the origin. Then the slope can be written simply as the ratio of forcelike to ratelike quantities:

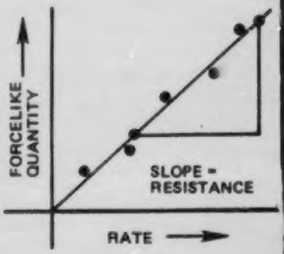
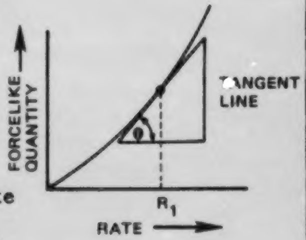


FIG. 18

$$\boxed{\text{RESISTANCE} = \frac{\text{FORCELIKE QUANTITY}}{\text{RATE}}}$$

There are times, however, when a graph of force versus rate will produce a curved graph. As indicated previously, in the case of turbulent fluid flow, this curve will occur because the frictional force then is proportional to higher powers of the velocity. The slope of the "line" will have different values at different points, which indicates that the resistance of the system is not a constant, but is changing continually. Resistance of the system can be found for any rate if the slope of the curve at the corresponding point (Figure 19) is found. Since the slope of a curve at a point on the curve is simply the slope of the tangent line to the curve at the point, resistance always can be found.



RESISTANCE OF SYSTEM WHEN OPERATING AT RATE R, IS THE SLOPE OF THE TANGENT LINE SHOWN.

FIG. 19

Table 2 contains a summary of the resistance in those systems in which resistance is dependent upon rate.

TABLE 2: RATE-DEPENDENT RESISTANCE

SYSTEM	FORCELIKE QUANTITY	RATE	RESISTANCE
Fluidal	Pressure difference Δp	Volume flow rate Q_V	$R_F = \frac{\Delta p}{Q_V}$
Electrical	Voltage ΔV	Current I	$R_E = \frac{\Delta V}{I}$
Thermal	Temperature difference ΔT	Heat flow rate Q_H	$R_T = \frac{\Delta T}{Q_H}$

DRY FRICTION

MATERIALS

Wooden block with hook and table top with smooth surface.
String.
Calibrated spring balance.
Set of weights (up to kilogram in size).

PROCEDURE

When a block is sliding at a constant speed, it is in equilibrium, and the forces on it must balance. Therefore, the pulling force exerted on the block to move it must be exactly balanced by the frictional force holding it back. On a horizontal surface, the normal force is the same as the weight of the block. From such a simple experiment, the coefficient of sliding friction between block and table can be determined.

1. Weigh wooden block; record this value in Data Table 1 as N.

2. Connect a string and spring balance to block as shown (Figure 20); slide block along table top at a uniform speed slow enough so that spring balance can be read. Spring balance registers frictional drag on block. Record this value as f.

3. Repeat this process several times, each time adding more weight to top of block.

4. Plot normal force N (x-axis) against frictional force f (y-axis), using data from table; draw a smooth line through these points.

5. Measure slope of line to determine coefficient of sliding friction between block and table top.

6. Use the following equation to calculate μ_k for each trial: $\mu_k = f_k/N$. Average results. How does the average value compare with answer to step 5?

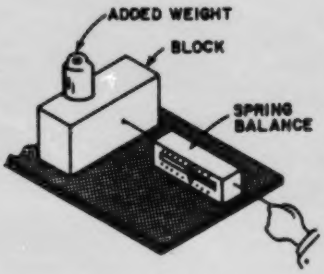


FIG. 20

DATA TABLE 1		
Sliding System	N (Total Weight Moved)	f (Spring Balance Reading)
Block alone		
Block with first weight added		
Block with second weight added		
Block with third weight added		
etc.		

FRICTION OF A BODY MOVING IN A FLUID

MATERIALS

Ball bearings approximately 6 mm diameter.
3 glass tubes at least 1 meter long and approximately 15 mm diameter.
Rubber stoppers for one end of each tube.
Motor oil and gear oil — 10w, 30w, and 90w.
Meter stick.
Stopwatch or other timer.
Vernier or micrometer calipers (metric units).

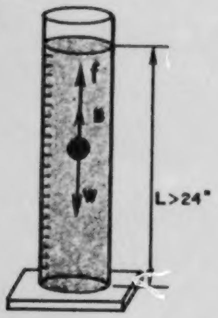


FIG. 21

PROCEDURE

An object falling freely in a fluid (Figure 21) reaches a terminal velocity, at which time it is in equilibrium. Upward forces balance the downward force. Downward force is the weight; upward forces are the buoyant force and the frictional drag of the fluid. This method will be used to measure frictional drag on a falling sphere for several different fluids.

1. Measure (in cm) ball bearing diameter with micrometer or vernier calipers. On a scale balance, determine their mass in grams. From this information, calculate volume ($V = 4/3\ \pi r^3$, where r is radius) and weight ($W = mg$) of bearing in cm^3 and dynes, respectively. Record all these results in Data Table 2.

2. Fill each cylinder with a different weight of oil (10w, 30w, and 90w); gently release ball bearing at surface for each different cylinder of oil; note how soon it reaches a terminal velocity. Make two marks on cylinder, as far apart as possible but within terminal velocity region of descent. Make a careful measurement of distance ℓ (in cm) between these marks with a meter stick.

3. Release ball bearing as before; time descent with a stopwatch, from first to second mark. Call this time t (seconds). Make at least three separate trials to measure t. Record these times with their average in Data Table 3.

4. Repeat for other two weights of oil supplied. Complete Data Table 3, calculating terminal velocity of descent for given ball bearing for each oil weight.

5. From given weight density (dynes per cm^3) of fluids used, calculate buoyant force on ball bearing in dynes for each fluid. Remember that —

   buoyant force = weight of displaced fluid

   = volume of ball bearing (cm^3) x weight density of fluid.

   Record in Data Table 4.

6. Calculate frictional force on ball bearing as it descends at terminal velocity in each weight of oil. Use fact that, in this case, forces are balanced so that: $f + B = W$, or $f = W - B$. Record in Data Table 4.

7. Determine resistance to motion offered by each oil. This resistance R will be ratio of frictional force f to terminal velocity of descent, v. $R = f/v$. Be sure to include proper units in results.

DATA TABLE 2

r (radius of ball bearing) = ____________ cm

V (volume of ball bearing) = (4/3) r = ____________ cm^3

m (mass of ball bearing) = ____________ grams

W (weight of ball bearing) = mass (m) x acceleration of gravity (g)

= ____________ grams x 980 cm/s^2

= ____________ dynes

DATA TABLE 3

ℓ = __________ cm.

		time (s)	average time (s)	velocity ($v = \ell/t_{av}$) (cm/s)
Fluid 1 10w	Trial 1			
	Trial 2			
	Trial 3			
Trial 2 30w	Trial 1			
	Trial 2			
	Trial 3			
Trial 3 90w	Trial 1			
	Trial 2			
	Trial 3			

DATA TABLE 4

	Weight Density of Fluid (d)	Buoyant Force (b=Vd)	Frictional Force (f=W-B)	Resistance (R = f/v)
Fluid 1 (10w oil)				
Fluid 2 (30w oil)				
Fluid 3 (90w oil)				

FLOW RESISTANCE OF A PIPE

MATERIALS

Large water tank (at least 5 gallon) with outlet near base.
Assorted glass tubing (with different bores and lengths) and matching bore rubber tubing.
Balance scale.
500-ml graduate.
Clamp for rubber tubing.

PROCEDURE

Resistance of a simple pipe to flow of water can be determined by measuring the rate of volume output for a given pressure differential.

1. Refer to Figure 22. Connect length of glass tubing to bottom outlet of water reservoir, using a short piece of rubber tubing. Place a clamp on rubber tube to stop water flow. Record length and bore diameter for glass tubing in Data Table 5.

2. Nearly fill reservoir, and measure water head h. Distance h measured in convenient length units represents pressure differential across tubing. Note: End of glass tube must be same height as outlet from reservoir.

3. With tubing in a horizontal position, open clamp to allow water to flow; catch outflow in a volume-calibrated container, such as a graduate. Time collection of this water with a stopwatch. Do not collect so much water that water head changes appreciably.

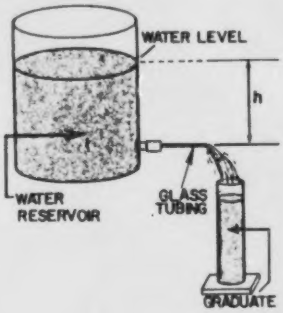


FIG. 22

4. Repeat for several (four or five) smaller values of pressure head h.

5. Repeat preceding steps for a second glass tube of different length and/or diameter bore.

6. From data taken, calculate volume flow rate Q_V; record in Data Table 5.

7. Plot on same graph sheet the data for tubing No. 1 first and then for tubing No. 2. Add pressure values (h) on vertical axis and volume flow rate on horizontal axis.

8. Determine slope of graph line for each tubing; record each value in proper units as resistance of tubing to fluid flow.

9. How are tubing dimensions related to its resistance?

DATA TABLE 5

	h	V	t	Q_V
First piece of tubing Length				
Diameter of bore				
Resistance from graph				
Second piece of tubing Length				
Diameter of bore				
Resistance from graph				

ELECTRICAL RESISTANCE

MATERIALS

d.c. low-voltage power supply.
Milliammeter, d.c. (500-ma range).
Voltmeter, d.c. (15-volt range).
Three 1/4-watt resistors, between 100 Ω and 1000 Ω.

PROCEDURE

Resistance of a circuit element to passage of an electric current is determined simply by the measurement of current when a known voltage is placed across resistance.

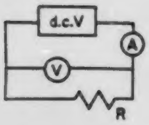


FIG. 23

1. Connect components of circuit as shown in circuit diagram (Figure 23). Be sure that voltmeter is connected directly across resistor, not across voltage supply unit. Use one of three resistors provided.

2. Vary voltage from zero to maximum value in approximately equal intervals, about eight steps in all, recording values of voltage and current each time, as read on meters. Record this data in Data Table 6.

3. Repeat for other two resistors.

4. Plot data for each resistor separately, but on one single sheet of graph paper. Place voltage values on vertical axis and current values on horizontal axis.

5. Determine resistance for each by measuring slope of each line. Remember that current must be expressed in amperes and potential difference in volts for the ratio V/I to give resistance in ohms. Record.

6. Compare results with manufacturer's value for resistors in Data Table 7.

DATA TABLE 6

R_1		R_2		R_3	
V (volts)	I (ma)	V	I	V	I

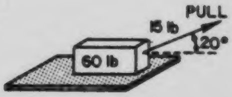
DATA TABLE 7

	Value from Graph	Manufacturer's Value
R_1		
R_2		
R_3		

SELF CHECK

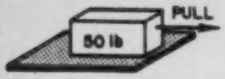
1. In a short paragraph, describe the concept of resistance and its effect in all energy systems. Include the final form of the energy expended to overcome resistance in all systems.

2. A 60-lb block can be pulled along at constant speed by a 15-lb force acting at an angle of 20° above the horizontal.



   a. How much of the 15 lb is lifting the block? How much is dragging it horizontally?
   b. What is the normal force — that is, the resultant force — pressing block and floor together.
   c. What is the coefficient of kinetic friction?

3. Only 12.5 lbs are required to pull a 50-lb crate at constant speed, but 15 lb to get it started. What are the coefficients of starting and of sliding friction?

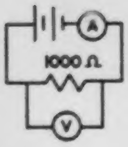


4. Describe differences between laminar and turbulent fluid flow; give examples of the importance of streamlining.

5. A garden hose has a pressure drop of 5 lb/in^2 along its length when the water flow rate is 3 gal/min. Determine the following:

   a. The fluid resistance of the pipe.
   b. The pressure drop if the flow rate is increased to 5 gal/min. Assume laminar flow.

6. The resistance of a hydraulic line is 1.5 lb/in^2/gal/min. Determine the flow rate when the pressure drop is 4.5 lb/in^2.



7. A 1000-ohm resistor is placed in the circuit shown. Assume that the resistance of the battery and ammeter are neglibible.

   a. What voltage is required to produce a reading of 1 ma by the ammeter A?
   b. What current is required to produce a reading of 6 volts by the voltmeter V?

8. When a 100-W light bulb is plugged into a 110V circuit, the current is 0.91A. What is the resistance of the light bulb?

9. A refrigerator must remove 2050 Btu's per hour from its interior to maintain an internal temperature of 35°F when the outside temperature is 95°F. Determine the following quantities:

   a. The thermal resistance of the refrigerator box.
   b. The thermal energy that must be removed each hour if the outside temperature is only 80°F.

10. The air frictional drag on a certain automobile is such that a force of 50 lb is required to keep it moving at a speed of 25 mi/h.

    a. Assume that the frictional force is proportional to the velocity. What is drag due to air resistance when the automobile is moving at 75 mi/h?
    b. Suppose that the automobile were poorly designed so that air friction varies as the square of the velocity. What is the drag due to the air at 75 mi/h in this case?

11. State the factors contributing to the following types of resistance and the effect each has upon resistance.

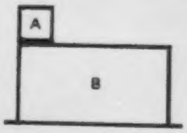
    a. Fluid resistance in long, straight pipes.
    b. Electrical resistance in wires.
    c. Thermal resistance in insulating materials.

12. Draw a graph showing resistance as the ratio of a forcelike quantity to a rate in the following energy systems:

    Fluidal.
    Electrical.
    Thermal.

13. State the fundamental difference between sliding friction and resistance in fluidal, electrical, and thermal systems.

14. Two blocks are arranged on a horizontal surface as shown at the right. What must be the acceleration of block B if block A is to start to slide on the top of block B? The coefficient of static friction between blocks A and B is 0.3. The mass of block A is 2 kg and the mass of block B is 8 kg.

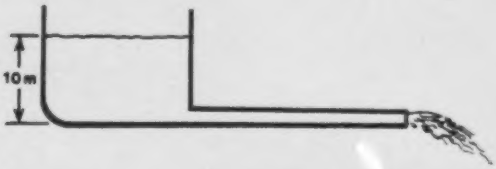


15. In a series circuit, the potential drop across a 100 Ω resistor is 2 V. What will be the potential drop across a 1 kΩ resistor which is in this circuit? (Hint: in a series circuit the same current flows through all the circuit elements.)

16. What force is required to push a 200 kg crate up a plane inclined at 20° if the coefficient of sliding friction between the crate and the inclined plane is 0.2?

17. What is the mass of water deposited in a swimming pool by a hose with a pressure drop of 10 lb/in^2 across its length and a fluid resistance of 0.3 $(\text{lb/in}^2)/(\text{ft}^3/\text{min})$ in a period of one hour?

18. Water flows out of a pipeline which is connected to a tank as shown below. If the pressure at the outlet at the end of the pipe is 1.90×10^5 N/m^2 when the height of water in the tank is 10 m what is the fluid resistance of the pipe if the volume flow rate is 0.2 m^3/minute?

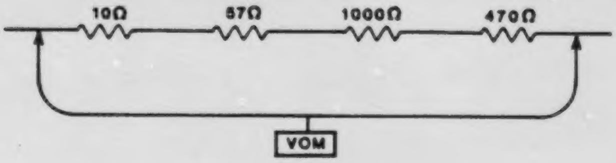


19. What is the thermal resistance of a calorimeter if the calorimeter's contents are maintained at a constant temperature of 40°C by electrical means which supply 5 calories/hr to the calorimeter? The temperature of the room in which the calorimeter is kept is 22°C.

20. What is the acceleration of a 40 kg wooden crate which is pushed horizontally across a stone floor by a horizontal force of 400 N? (See Table 1.)

21. A block rests on an inclined plane. The coefficient of static friction between the block and the plane is 0.2. What is the maximum angle of inclination of the plane for which the block will not slide down the plane?

22. A voltmeter placed across the series arrangement of resistors shown below reads 6 V. What is the current flowing through the resistors?



23. What is the difference in the magnitudes of the coefficients of static and kinetic friction between a block and a table if the same force which is required to just start the block in motion accelerates the block 0.1 m/s^2 once the block starts moving?

24. What drag force is necessary to reduce the downward acceleration of a 150 kg skydiver to 1 m/s^2?

25. An experimentalist is modeling resistance to turbulent flow by using the relationship,

$$R_F = \frac{\Delta P}{(Q_H)^n} .$$

Where n is an integer. Find the most probable value of n given the following data. Assume ΔP is constant throughout the experiment.

Trial	R_F $\left(\frac{lb/in^2}{gal/min}\right)$	Q_H (gal/min)
1	0.14	27
2	0.20	24
3	0.46	18

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POWER

CHAPTER 6-0

A major problem facing our nation today is the energy crisis. Energy from petroleum products is being consumed faster than new supplies can be found. On the one hand, the high rate of energy consumption in this country has led to many desirable outcomes. For instance, we have the largest gross national product in the world, which means that the United States is a prosperous nation. The gross national product is related very closely to the rate of energy consumption. When the rate of energy consumption is high, factories are approaching full production. As the rate of consumption decreases, so does the production. On the other hand, the high rate of energy consumption could exhaust the world's oil and coal supplies before an alternate source is available.

Thus, the rate of energy consumption is an important issue in economics and in politics; and as energy costs rise, it becomes a more important issue to most individuals. The rate of energy consumption, called "power," is also of great importance in science and technology. This module discusses the concept of power and its applications in mechanical, fluidal, electrical, and thermal energy systems.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define "power" as it applies, in general, to all energy systems; and write equations that relate work, elapsed time, force, and rate to power in these energy systems:

    Mechanical system.
    Fluidal system.
    Electrical system.

- List for each energy system the SI and English units used to define power.

- Given any two of the following quantities in any energy system, determine the third:

  Work (or Forcelike Quantity x Displacementlike Quantity).
  Elapsed time.
  Power.

- Given any two of the following quantities in any energy system, determine the third:

  Forcelike quantity.
  Rate.
  Power.

- Define the following terms:

  Input power.
  Output power.
  Efficiency.

- Given any two of the following quantities in any energy system, determine the third:

  Input power.
  Output power.
  Efficiency.

- Convert values for power expressed in any energy unit to equivalent values expressed in any other unit.

POWER EQUATIONS

"Power" is defined as "the rate at which energy is transferred or work is done." Equation 1 is the mathematical expression defining power.

$$P = \frac{W}{t} \qquad (1)$$

where
P = power.
W = work.
t = elapsed time.

In Module 2-0, Work," work was found to be the product of a forcelike quantity and a displacementlike quantity. Thus, Equation 1 can be written for any energy system as Equation 2.

$$P = \frac{FD}{t} \qquad (2)$$

where
F = forcelike quantity.
D = displacement like quantity.

In Module 3-0, "Rate," rate was defined as a displacementlike quantity divided by elapsed time. Equation 2, thus, can be written in the form of Equation 3.

$$P = FR \qquad (3)$$

where
P = power.
F = forcelike quantity.
R = rate.

The two unifying equations for power in all energy systems are Equations 1 and 3. They can be stated, respectively, as follows:

$$\boxed{\text{POWER} = \frac{\text{WORK}}{\text{TIME}}}$$

$$\boxed{\text{POWER} = \text{FORCELIKE QUANTITY X RATE}}$$

The application of these two equations in mechanical, fluidal, and electrical energy systems is presented later in this module.

EFFICIENCY

Energy is not created by any system, but often is transferred from one system to another or transferred within a given system. Energy cannot be transferred without loss of a portion of it. Resistance and friction, which produce heat, are the most frequent causes of energy loss. Clearly, in order to operate any system, energy must be supplied to the system at a rate that equals the rate of energy loss plus the rate at which useful work is performed.

The rate at which energy is transferred, or at which work is performed, is defined as "power." "Power input" means the rate at which energy is transferred into a system, while "power output" is the rate at which the system performs work. Figure 1 demonstrates this concept in general terms.

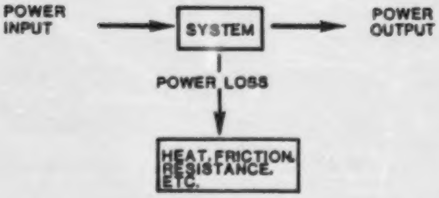

POWER INPUT
SYSTEM
POWER OUTPUT
POWER LOSS
HEAT, FRICTION, RESISTANCE, ETC.

FIG. 1

The ratio of power output to power input in any system is referred to as "efficiency." The quantity usually is expressed as a percentage and can be calculated from Equation 4:

$$\eta = \frac{P_{out}}{P_{in}} (100) \qquad (4)$$

where

η = percent efficiency.
P_{out} = output power.
P_{in} = input power.

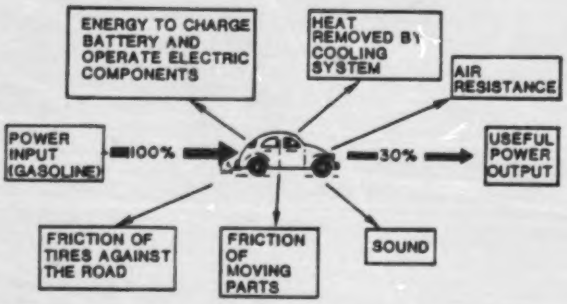

ENERGY TO CHARGE BATTERY AND OPERATE ELECTRIC COMPONENTS
HEAT REMOVED BY COOLING SYSTEM
AIR RESISTANCE
POWER INPUT (GASOLINE)
100%
30%
USEFUL POWER OUTPUT
FRICTION OF TIRES AGAINST THE ROAD
FRICTION OF MOVING PARTS
SOUND

FIG. 2

A typical automobile operates at an efficiency of about thirty percent, which means that only 30% of the energy available in gasoline actually is transferred into motion of the automobile. A few of the major causes of this poor efficiency is shown in Figure 2. Of course, the air conditioning and other electric components utilize energy, which usually is considered to be "useful power output."

The efficiency of energy conversion devices is of great importance in the production and application of energy. Efficiency will be calculated with Equation 4 in several examples in this module.

MECHANICAL SYSTEMS

TRANSLATIONAL

In linear mechanical systems, work is the product of force and displacement. When these quantities are substituted into Equation 1, the result is Equation 5.

$$P = \frac{Fd}{t} \tag{5}$$

where
F = force.
d = displacement.

When velocity (rate) and force are substituted into Equation 3, Equation 6 results.

$$P = Fv \tag{6}$$

In all energy systems, the appropriate units of power are obtained by dividing the energy units by the time units or by multiplying the rate units by the forcelike units. In the SI system, the units from Equation 5 are (N•m)/s, and the units from Equation 6 are N•(m/s). Thus, either method results in power units of newton-meters per second. A newton-meter (N•m) is defined as a joule (J) of energy, and power sometimes is expressed as joules per second (J/s). In the SI system, this unit is called a "watt" (W). When the same procedure is applied to the English system of units, the result is power units of foot-pounds per second (ft•lb/s).

The English system of units has developed over a long period of history. Whenever a convenient unit was needed, it was invented. Experiments dealing with power were performed during a time when horses commonly were used, and the power of machines was compared with that of horses. An "average horse" could perform work at a rate of 550 ft•lb per second. This relationship still is in use today, as we define the unit of a horsepower to be equivalent to 550 ft•lb/s:

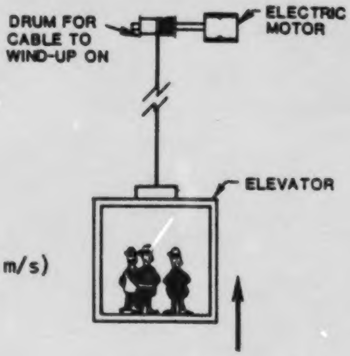
$$1\ \text{hp} = 550\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}$$

Example A illustrates the use of Equations 5 and 6 with SI units.

EXAMPLE A: POWER OF AN ELEVATOR

Given: An elevator with a mass of 1200 kg moves upward at a constant velocity of 2 m/s.

Find: Power that the elevator motor must produce.



Solution: Use Equation 6:

$$\begin{aligned} P &= Fv = mgv \\ &= (1200\ \text{kg})(9.8\ \text{m/s}^2)(2\ \text{m/s}) \\ &= (11760\ \text{N})(2\ \text{m/s}) \\ P &= 23520\ \text{N}\cdot\text{m/s} \end{aligned}$$

Use Equation 5:

If the elevator travels at an average speed of 2 m/s, it is displaced 2 m in 1 s.

$$P = \frac{Fd}{t}$$

$$= \frac{(11760\ \text{N})(2\ \text{m})}{(1\ \text{s})}$$

$$P = 23520\ \text{N}\cdot\text{m/s}$$

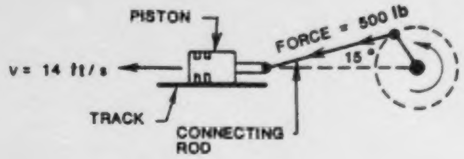
The power of the elevator is 23.52 kW. Note that the following are equivalent power units in the SI system:

$$\text{W} = \frac{\text{J}}{\text{s}} = \frac{\text{N}\cdot\text{m}}{\text{s}} = \frac{\text{kg}\cdot\text{m}^2}{\text{s}^3}$$

Power is not a vector quantity, although both force and velocity are vectors. Thus, in order for Equation 6 ($P = Fv$) to be valid, the effective component of force and the velocity must be measured in the same direction. A vector force problem often must be worked to determine the force component that is parallel to the velocity of the moving object. Example B illustrates this idea and the use of English units in solving a problem.

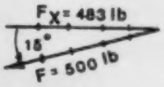
EXAMPLE B: POWER DELIVERED TO A PISTON

Given: A movable piston is being forced by a connecting rod to move along a straight path. The connecting rod makes an angle of 15° with the line of motion of the piston at the instant that the applied force is 500 lb and that the piston's velocity is 14 ft/s.



Find: The instantaneous power delivered to the piston.

Solution:



Graphical solution shows that force component parallel to motion of piston is 483 lb.

$$
\begin{aligned}
F_x &= F \cos \theta \\
&= (500\ \text{lb}) \cos (15°) \\
&= (500\ \text{lb})(0.966) \\
F_x &= 483\ \text{lb}
\end{aligned}
$$

$$
\begin{aligned}
P &= Fv \\
&= (483\ \text{lb})\ (14\ \text{ft/s}) \\
&= 6762\ \frac{\text{ft}\cdot\text{lb}}{\text{s}} \\
&= \left(6762\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}\right) \left(\frac{1\ \text{hp}}{550\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}}\right) \\
P &= 12.3\ \text{hp}
\end{aligned}
$$

ROTATIONAL SYSTEMS

In rotational mechanical systems, the Equations for power are —

$$P = \frac{\tau\theta}{t} \tag{7}$$

where
τ = torque.
θ = angle of rotation.

and —

$$P = \tau\omega \tag{8}$$

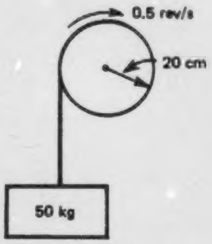
where
ω = angular velocity.

The units of rotational mechanical power are the same as those of linear mechanical power. Examples C and D show the use of Equations 7 and 8, respectively.

EXAMPLE C: POWER OF A WINCH

Given: An electric winch is used to lift a 50 kg mass. The radius of the winch drum is 20 cm, and it turns one-half a revolution in one second. The motor driving the winch has an electrical input power of 385 W.

Find:
(a) Output power of winch.
(b) Efficiency.



Solution: (a) Torque:

$$\tau = F\ell = mg\ell$$
$$= (50\ \text{kg})(9.8\ \text{m/s}^2)(0.2\ \text{m})$$
$$= 98\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$
$$\tau = 98\ \text{N}\cdot\text{m}$$

Notice that N•m is a unit of torque in this case.

Angle of rotation:

$$\theta = (0.5\ \text{rev})\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)$$
$$\theta = 3.14\ \text{rad}$$

Power:

$$P = \frac{\tau\theta}{t}$$

$$= \frac{(98\ \text{N}\cdot\text{m})(3.14)}{1\ \text{s}}$$ "radians" is dropped.

$$= 307.7\ \frac{\text{N}\cdot\text{m}}{\text{s}}$$

$$P = 307.7\ \text{W}.$$

(b) Efficiency:

$$\eta = \frac{P_{out}}{P_{in}}\ (100)$$

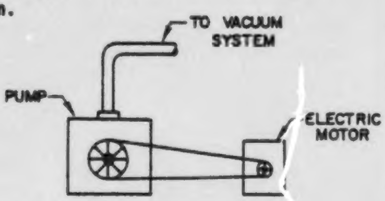
$$= \frac{307.7\ \text{W}}{385\ \text{W}}\ (100)$$

$$\eta = 80\%$$

EXAMPLE D: POWER OF A VACUUM PUMP

Given: A vacuum pump requires a torque of 35 ft·lb and has a rotational rate of 750 rpm.

Find: The minimum power rating for a motor to drive the pump.



Solution: In Equation 8 ω must be expressed in units of rad/s.

$$\omega = \left(750\ \frac{\cancel{\text{rev}}}{\cancel{\text{min}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\ \text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \cancel{\text{rev}}}\right)$$

$$= 25\ \pi\ \text{rad/s}$$

$$\omega = 78.5\ \text{rad/s}$$

$$P = \tau\omega$$

$$= (35\ \text{ft}\cdot\text{lb})(78.5/\text{s})$$

$$= 2750\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}$$

$$= \left(2750\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}\right)\left(\frac{1\ \text{hp}}{550\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}}\right)$$

$$P = 5\ \text{hp}$$

FLUIDAL SYSTEMS

In fluidal systems, the power requied to push a liquid through a pipe at a given rate is determined by Equation 9 or Equation 10.

$$P = \Delta p \frac{\Delta v}{\Delta t} \qquad (9)$$

where
Δp = pressure difference.
Δv = volume displaced.
Δt = elapsed time.

$$P = Q\Delta p \qquad (10)$$

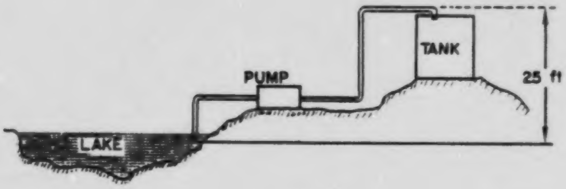
where
Q = volume flow rate.

The units of power are the same as for mechanical systems. Example E illustrates the use of Equations 9 and 10 in solving a fluid problem.

EXAMPLE E: POWER IN A FLUID SYSTEM

Given: A pump must fill a 5000-ft^3 tank located 25 ft above the level of the lake, as shown; and the tank is to be filled in 15 minutes. The weight density of water is 62.4 lb/ft^3.

Find: The power that must be delivered by the pump.



Solution: Pressure head:

$$\Delta p = \rho_w h$$

$$= (62.4\ \text{lb/ft}^3)(25\ \text{ft})$$

$$\Delta p = 1560\ \text{lb/ft}^2$$

Power from Equation 9:

$$P = \Delta p \frac{\Delta v}{\Delta t}$$

$$= \frac{(1560\ \text{lb/ft}^2)(5000\ \text{ft}^3)}{(15\ \text{min})(60\ \text{s/min})}$$

$$P = 8667\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}$$

Volume flow rate:

$$Q = \frac{\Delta v}{\Delta t}$$

$$= \frac{5000\ \text{ft}^3}{900\ \text{s}}$$

$$Q = 5.556\ \text{ft}^3/\text{s}$$

Power from Equation 10:

$$P = Q\Delta p$$

$$= (5.556\ \text{ft}^3/\text{s})1560\ \text{lb/ft}^2)$$

$$= 8667\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}$$

$$P = 15.7\ \text{hp}$$

ELECTRICAL SYSTEMS

In an electrical system, the power required to move a charge through a potential difference in a given time can be determined by use of Equation 11.

$$P = \frac{QV}{t} \tag{11}$$

where

Q = charge transferred.
V = potential difference (voltage).

Since current is defined as "the rate of charge flow" (Module 3-0, "Rate"), this equation can be rewritten as Equation 12.

$$P = IV \tag{12}$$

where
I = electric current.

In this equation, current is measured in amperes, and potential difference is measured in volts. Power is expressed in watts. Thus, one volt-amp is equal to one watt. Example F illustrates the use of these units and of Equation 12 to solve a problem.

EXAMPLE F: CURRENT REQUIRED BY A WATER HEATER

Given: An electric water heater operates at 240V and is rated at 1500 W.

Find: Current required.

Solution:

$$P = IV$$

$$I = \frac{P}{V}$$

$$= \frac{1500\ \text{W}}{240\ \text{V}}$$

$$I = 6.25\ \text{A}$$

Relationships other than P = VI often are used to determine power in electrical systems. These equivalent relationships are developed by substituting Ohm's law into P = VI, as shown below:

$$P = VI$$

$$I = \frac{V}{R}\ \text{(Ohm's Law)}$$

Therefore, substituion for I yields the following:

$$P = V\left(\frac{V}{R}\right)$$

or

$$P = \frac{V^2}{R} \tag{13}$$

and

$$P = VI$$

$$V = IR\ \text{(Ohm's law)}$$

Therefore, substitution for V yields this result:

$$P = (IR)\ I$$

or $$P = I^2R \qquad (14)$$

In electrical systems, the power always is measured in watts; there are no English units for electrical power. Thus, the conversion of SI units to English units often is necessary when dealing with problems involving both electrical and mechanical power. The conversion factor is —

$$746\ \text{W} = 1\ \text{hp}.$$

Example G shows the use of this conversion factor and of Equation 12 in determining the efficiency of an electric motor.

EXAMPLE G: EFFICIENCY OF AN ELECTRIC MOTOR

Given: A 1/4 hp electric motor operates on a 24-volt source and requies a current of 9.47 A.

Find: Efficiency of the motor.

Solution: Input power:

$$\begin{aligned} P_{in} &= IV \\ &= (9.47\ \text{A})(24\ \text{V}) \\ P_{in} &= 227\ \text{W} \end{aligned}$$

Output power:

$$P_{out} = (0.25\ \text{hp})\left(\frac{746\ \text{W}}{1\ \text{hp}}\right)$$

$$P_{out} = 186.5\ \text{W}$$

Efficiency:

$$\begin{aligned} \eta &= \frac{P_{out}}{P_{in}}\ (100) \\ &= \frac{(186.5\ \text{W})}{(227\ \text{W})}\ (100) \\ \eta &= 82\% \end{aligned}$$

THERMAL SYSTEMS

In thermal systems, the displacementlike quantity is heat energy, and the rate formula has the units of heat energy divided by time. Thus, heat flow rate also is the thermal power. The general power equation — power equals forcelike quantity times rate — is not applied to thermal systems. Thermal power usually is expressed in Btu/h in the English system of units and cal/s in SI units.

SUMMARY

The concept of power has useful meaning in many energy systems. In this module, the idea of power is applied to common problems in electrical, fluid, mechanical rotational, and mechanical translationsal systems. Power is the rate of energy consumption and in a given energy system is the product of the forcelike quantity and rate appropriate to that system. Table I summarizes this principle as applied to various energy systems.

TABLE 1: SUMMARY OF VARIOUS ENERGY SYSTEMS

System	Forcelike Quantity	Ratelike Quantity	Power (Common Units)
Mechanical-translational	Force (F)	Velocity (v)	$P = Fv$ (ft•lb/s, hp, N•m/s)
Mechanical-rotational	Torque (τ)	Angular Velocity (ω)	$P = \tau\omega$ (ft•lb/s, hp, N•m/s)
Fluid	Pressure (p)	Volume flow rate (Q_v)	$P = p\,\Delta V/\Delta t$ (ft•lb/s, hp, N•m/s)
Electrical	Voltage (V)	Current (I)	$P = VI$ (w)

CLIMBING STAIRS

MATERIALS

Isolated stairway.
Tape measure.
Stopwatch.

PROCEDURE

The following procedures illustrate the use of the concept of power in different energy systems.

Determine the power that an individual develops when running upstairs.

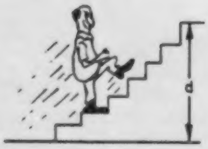


FIG. 3

1. Perform this experiment on a safe stairway, reasonably free of other traffic (Figure 3).
2. Record individual's weight in Data Table 1. Determine and record vertical height d of the stairs.
3. Start stopwatch at same instant stairs are climbed at a rapid but safe speed. Record time required to climb stairs.
4. Calculate average vertical speed and power developed in climbing stairs; calculate in units of ft•lb/s, hp, and watts.

DATA TABLE 1

Measure —

Vertical height of stairs (d) ________________ ft

Weight (F) ___________ lb

Time to climb stairs (t) ________ seconds

Calculate —

Average vertical speed up the stairs $(d/t) = v_{av} =$ ________

Average power in ft•lb/s $(P = Fv_{av})$ ____________

Average power in hp $(1 \text{ hp} = 550 \frac{\text{ft} \cdot \text{lb}}{\text{s}})$ ____________

Average power in watts (1 hp = 746 watts) ____________

LIFTING A WEIGHT

MATERIALS

Mass — approximately 2 kg.
String.
Wooden dowel (1 in diameter or larger and 10 in long).
Meter stick.
Stopwatch or laboratory timer.

PROCEDURE

In this experiment, a weight is lifted by attaching it to a string and winding the string on a wooden rod, as shown in Figure 4. Measurement of the time taken to lift the 2-kg weight will make possible the calculation of power expended in the process. Use constant units (English or metric) throughout.

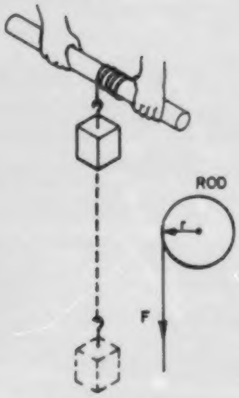


FIG. 4

1. Attach weight to one end of string, and fasten other end with a small tack at center of rod, as shown. Hold rod so that string is taut and weight barely touches ground.

2. Use a stopwatch or a lab timer to determine time required to rotate the rod and, thus, to wind the string and to lift the mass. Try to rotate road at a constant angular speed. Record this time in Data Table 2.

3. Measure diameter of rod; calculate, record radius of rod.

4. Count number of loops of string wound on stick, estimating any fractional part of a complete loop. This represents the number of rotations of stick.

5. Complete Data Table 2, calculating power required to raise the mass in time given.

DATA TABLE 2

Weight lifted = force of string on rod (F) = ________________

Time to lift weight (t) = ________________

Radius of rod (r) = ____________________

Number of loops wound on rod (n) = ______________________

Net angular displacement of rod in radians (θ) = $2\pi n$ = ________

Power expended in lifting the weight, or $P = \frac{\tau\theta}{t} = \frac{FR\theta}{t}$ = _______

ELECTRIC PUMP

MATERIALS

Electric pump (low-voltage a.c. or d.c.).
Ammeter.
Voltmeter.
Two large containers of known volume.
Connecting hose.
Low-voltage power supply to match specifications of the electric pump, a.c./d.c.

PROCEDURE

This experiment requires power calculations at both input and output of a combined electrical-fluid system.

1. Arrange an electric pump to lift water from a reservoir on floor to a higher level, as shown in Figure 5.

2. Connect an ammeter and voltmeter in pump circuit, as shown in Figure 6. Measurements of voltage and current to pump motor during operation allows calculation of electrical power input.

3. With a stopwatch, measure time necessary for pump to fill a relatively large container of known volume. Measure voltage V, current I, and liquid head h. Record all data in Data Table 3.

4. Complete Data Table 3, calculating power input and useful power output of system. (Examples D and F may be helpful in these calculations.)

5. Calculate the overall efficiency of the system.

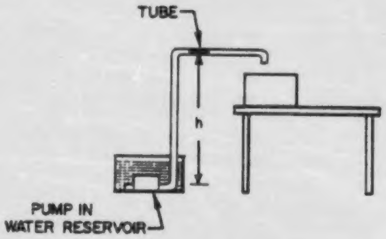


FIG. 5

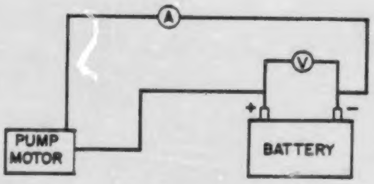


FIG. 6

DATA TABLE 3	
Power Input (Electrical System)	Power Output (Fluid System)
Voltage across pump V = ________ Current through pump I = ________ Power input to pump = VI = ______	Weight density of water ρ = ________________ Pressure head h = ________ Pressure $p = \rho h$ = ________ Volume of water pumped ΔV = ________ Time required at Δt = ______ Use power output in transferring water: $P = \Delta p \frac{\Delta v}{\Delta t}$ = ____________
Efficiency (%) = $\frac{\text{power output}}{\text{power input}} \times 100$ = ________________	

SELF CHECK

1. Define power in a general way that applies to all energy systems.

2. Write two equations for power in each of the following energy systems:

    a. Mechanical translational.
    b. Mechanical rotational.
    c. Fluidal.
    d. Electrical.

3. A force of 250 lb is applied to push a wagon a distance of 200 ft in 2 minutes. Determine the horsepower required.

4. An automobile engine develops 50 hp at a rotational rate of 2000 rpm. How much torque is produced by the engine?

5. A 5-hp (output) water pump has an electrical input power of 5 kW. It is used to fill a 2500-ft^3 tank and must raise the water through a height of 46 ft. Determine the efficiency of the pump and the time required to fill the tank. (The weight density of water is 62.4 lb/ft^3.)

6. Calculate the efficiency of a system for which an electric motor is used to turn a flywheel attached to a load if the motor draws 7.5 amps when connected to a 240-volt line.

The flywheel requires a constant torque of 42 ft·lb to maintain a constant angular velocity of 250 r/min. (1 ft·lb/s = 1.356 watts.)

7. A current of 10 A flows through a 2 Ω resistor for 1 min. The conversion of electrical energy to thermal energy has an efficiency of 100%. How much heat energy is produced? (1 cal = 4.186 J.)

8. Given the following conversion factors, determine the conversion factor for watts to Btu/h:

   1 Btu = 778 ft·lb.
   1 hp = 746 W.
   1 hp = 550 $\frac{\text{ft}\cdot\text{lb}}{\text{s}}$.

9. What is the mass of the load which a 1200 kg elevator carries upwards with a constant velocity of 3.5 m/s if the elevator motor produces 50,000 watts of power?

10. What is the efficiency of an elevator motor which operates on 600 volts and draws a current of 15 amps if the motor can lift an elevator of mass 600 kg at a constant speed of 1 m/s?

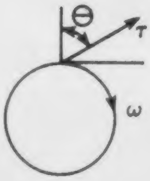
11. Forty m^3/sec of water falling from a height of 30 meters are used to turn a flywheel. If 80 percent of the water's power is delivered to the flywheel and the torque on the flywheel is 20,000 N·m, what is the constant angular velocity of the flywheel?

12. How high above a reservoir should a 2000 m^3 tank be placed if a motor which delivers 20,000 watts is to fill the tank in 40 minutes?

13. An electric winch is being designed to lift a block of mass M at a constant speed V. The winch is driven by a motor which has an input power P. Show that the efficiency of the winch does not depend on the winch's radius.

14. What is the resistance of a given resistor if a current of 30 milliamperes flowing through the resistor dissipates 5×10^{-3} calories of heat per second?

15. A constant torque of 20 N·m is applied to the rim of a flywheel as shown below. At what angle θ is the torque applied if the flywheel rotates with an angular speed of 10 rad/s and has an output power of 180 watts?



16. How much charge can be moved from a reservoir at -3 volts with respect to ground to one at +8 volts with respect to ground in 1 minute if the power required is 40 watts?

17. How many horsepower are required to push a 20 lb wagon up a hill of length 20 ft which makes an angle of 30° with the horizontal in 20 seconds? Assume there is no friction between the wagon and the hill. Also assume that the wagon is pushed up the hill with a constant velocity.

18. Does it take more power to lift an elevator at constant velocity than it does to lower the elevator at the same constant velocity? Explain.

19. What should be the operating voltage of a motor if it is to lift 20 kg through a distance of 40 meters in 20 seconds? The motor draws a current of 0.5 A and is 70% efficient.

20. How fast does a piston move if a force of 20 N which makes an angle of 45° with the direction of the piston's motion drives the piston, and the piston delivers a power of 200 watts?

21. A portable heater has a power output of 100 watts. In each hour of operation the heater reduces the heating bill by 2¢. If the electricity costs 5¢ per kilowatt hour and the heater is 67% efficient, how many hours of operation will it take for the heater to pay for itself? The heater cost is $100.

22. What is the current flowing through a 1 kΩ resistor, if when the resistor is immersed in a calorimeter filled with 100 grams of water, the temperature of the water increases by 0.2 C° per second? The calorimeter has a water equivalent of 100 cal/°C.

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POTENTIAL AND KINETIC ENERGY

CHAPTER 7-0

Concept Module 2-0, "Work," described work as the product of a displacement like quantity and a forcelike quantity. Energy was defined as "the ability to do work." Energy can be contained in a system in a variety of forms. Most of the world's energy needs are supplied today by the burning of fossil fuel to convert their stored chemical energy to thermal energy. Solar energy is electromagnetic radiation emitted by the sun. This module discusses two important forms of energy in technology and demonstrates farther the unificiation of physics. Kinetic energy is the energy of moving mass; it is understood most easily in mechanical and fluidal systems, although heat energy actually is the kinetic energy of moving molecules. Potential energy is the energy that exists due to position or potential difference. Potential energy involves no motion, but it has the capacity to produce motion at some later time. The Principle of the Conservation of Energy states, "Energy can be neither created or destroyed." The total energy in any closed system remains constant, although it may change in form. This module describes potential and kinetic energy and discusses the conservation of energy in mechanical, fluidal, electrical and thermal systems.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define potential energy, kinetic energy, and conservation of energy by using examples from mechancial systems.
- Given any two of the quantities in the following groups, determine the third:

Mass, velocity, kinetic energy.
Mass, height, potential energy.
Spring constant, spring displacement, potential energy.
Moment of inertia, angular velocity, kinetic energy.
Capacitance, voltage, potential energy.

- Given Bernoulli's Equation and the height of liquid in a tank, determine the exit velocity at the bottom of the tank if there is no fluid friction.

- List and describe the three processes that transfer thermal energy.

- Discuss the conservation of energy as it applies to fluidal, electrical, and thermal systems.

DISCUSSION

Module 2-0, "Work," presented the concept that work done on an object represents the amount of energy transferred to that object. Work was defined in each energy system as "the product of a forcelike quantity and a displacementlike quantity." Work is done when a forcelike quantity acts to cause motion in a system. Energy is the ability to do work. The presence of energy in a system does not mean that forcelike quantities are producing changes, but only that there is a potential for that to occur in the future.

No work is done unless energy is transferred. The energy may change forms, as when friction changes mechanical energy into thermal energy; or it may retain the same form, as when a moving object strikes a stationary one, causing it to move. In all cases, the total energy of the system is conserved.

MECHANICAL SYSTEMS

TRANSLATIONAL

In many cases, the work done on an object appears primarily as energy of motion (kinetic energy). Total kinetic energy of a moving object is related to its mass and velocity by Equation 1. The sample problem (Example A) illustrates the use of Equation 1.

$$E_K = 1/2\ mv^2 \qquad (1)$$

where

E_K = kinetic energy.
m = mass.
v = velocity.

EXAMPLE A: KINETIC ENERGY OF A MOVING BALL

Given: A 250-gram ball moves with a velocity of 15 m/s.

Find: Amount of kinetic energy it possesses.

Solution: From Equation 1 —

$E_K = 1/2\ (0.25\ \text{kg})(15\ \text{m/s})^2$

$E_K = 28.1$ joules.

$E_K \geq (\text{kg})(\text{m/s})^2$

$\geq \left(\frac{\text{kg} \times \text{m}}{\text{s}^2}\right)(\text{m})$

NOTE: When only units are substituted into the equation, kinetic energy is shown to have the same units as work (i.e., Joules).

(Recall: $\frac{\text{kg}\cdot\text{m}}{\text{s}^2}$ = newton (N).)

$\geq N \times m$

$E_K \geq$ joules

When work is one to lift an object, the work becomes potential energy of the object (Figure 1).

To lift the box to the shelf in Figure 1, the man must exert a force equal to the weight of the box. We know that the work done by the man is equal to Wh (where W = weight, and h = height from floor to shelf). This value is the exact amount of increase in potential energy of the box after it is placed on the shelf. When the box is placed on the shelf, the gravitational potential energy of the box with respect to the floor is expressed by Equation 2.

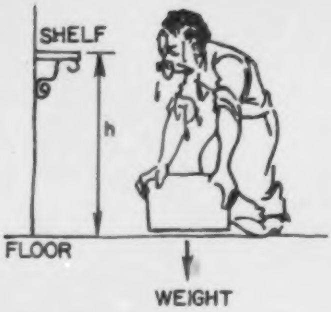


FIG. 1

$$E_P = Wh \qquad (2)$$

where
E_P = potential energy.
W = weight.
h = height of shelf from floor.

Since weight of an object is related to its mass by the gravitational constant according to $W = mg$, Equation 2 often is given in the form of Equation 3.

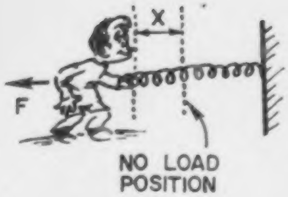


FIG. 2

$$E_P = mgh \qquad (3)$$

where
g = gravitational constant.

Gravitational potential energy is an important and useful concept. There are means of storing energy other than the raising of weight; for example, a stretched or compressed spring waiting to be released possesses potential energy.

Work necessary to stretch the spring beyond its "no load" position (Figure 2) is given by $W = Fx$. In this case, F is not a constant force, but increases as x increases.

Potential energy of the spring stretched x units beyond its normal length is given in Equation 4.

$$E_p = \frac{1}{2} kx^2 \qquad (4)$$

where

k = spring constant (depends upon the mechanical stiffness of the spring; see below).

x = units to which spring is stretched beyond its normal limits.

The work put into the spring (Work = Fx) is stored by the stretched spring as potential energy ($E_p = 1/2\ kx^2$). The value of k is just the magnitude of the force required for unit stretch of the spring; for example, 5 lb/inch. For a given spring, the value of k does not vary, provided the spring is not stretched beyond its elastic limit (i.e., the point at which the spring will not return to the no-load position if force is removed).

Concept Module 5-0, "Resistance," shows that resistance opposes motion and eventually causes moving objects to slow down or stop. Resistance cannot be completely eliminated from a system, even with the best lubrication. However, friction often is omitted, as a matter of convenience, when energy systems are studied. Friction will be neglected, therefore, throughout the remainder of this module. Obviously, the conclusions reached are only good approximations of "real world" situations in which friction plays a large part.

When no energy is lost to friction, the sum total of the kinetic and potential energy of a system remains constant. Thus, if kinetic energy of the system decreases, the potential energy of that system increases an equal amount. This concept is amplified in Example B.

EXAMPLE B: POTENTIAL ENERGY OF STORED WATER

Given: Two tons of water are at the top of a 200-foot waterfall, ready to go over the edge.

Find: (a) The amount of potential energy that the water possesses with respect to the bottom of the falls, (b) the amount of potential and kinetic energy the water has when it has fallen halfway down and (c) the amount of kinetic energy the water has when it has reached the bottom.

Solution: (a) At top of falls —

$$E_p = mgh = Wh$$
$$= (4000\ \text{lbs})(200\ \text{ft})$$
$$E_p = 800{,}000\ \text{ft}\cdot\text{lbs}$$
$$E_K = 0$$

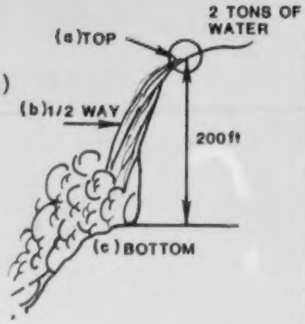
(b) At halfway point —

$$E_P = (4000 \text{ lbs})(100 \text{ ft})$$
$$= 400{,}000 \text{ ft}\cdot\text{lbs}$$

$$E_K + E_P = 800{,}000 \text{ ft}\cdot\text{lbs}$$
$$\therefore E_K = 400{,}000 \text{ ft}\cdot\text{lbs}$$

(c) At bottom —

$$E_P = 0$$
$$E_K = 800{,}000 \text{ ft}\cdot\text{lbs}$$



We have assumed that the sum of $E_K + E_P$ is the same throughout.

CONSERVATION OF ENERGY

The fact that the sum of potential and kinetic energy remains unchanged, even while there is a transformation of one form of energy to the other, is a demonstration of the Principle of the Conservation of Energy. This principle is one of the cornerstones upon which all of science and technology is built.

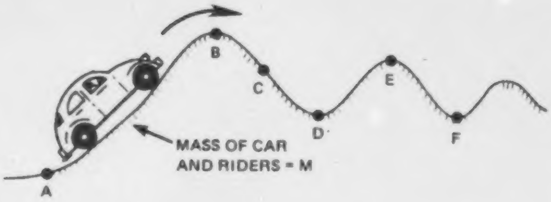


FIG. 3

Consider the changes in kinetic and potential energy of a roller coaster car as it travels on the track illustrated in Figure 3.

From point A up to B, energy must be supplied to the car by an electric winch. Assume the winch is released at B. At point B, energy supplied by the electric winch has been stored in the car as potential energy. This energy is equal to mgh, as given by Equation 3. As the car coasts down the first hill past point C toward D, its energy is changing rapidly from potential to kinetic. As kinetic energy increases, so does the car's velocity (speed), according to Equation 1 ($E_K = 1/2\ mv^2$). The car always is moving faster at low points on the track (points D and F) and slower at high points (B and E). However, at any point the sum total of its kinetic and potential energy is equal to the original amount of energy supplied to the car during ascent from A to B. Friction, if considered, gradually steals some of the energy away.

A spring with an attached mass bouncing from its end is another example of a system with changing kinetic and potential energy (Figure 4).

Work is done on the system in Figure 4 when the spring (with mass attached) is stretched from position A to B, a distance equal to x. The amount of energy necessary to do this work is given by Equation 4 ($E_p = 1/2\ kx^2$). When the spring is released, it moves upward through the rest position. At the rest position, the displacement x is zero; and potential energy $1/2\ kx^2$ equals zero at this point. However, as the mass passes the rest point, it is moving with its greatest speed and, thus, has its greatest kinetic energy. As the mass rises above its rest position, its speed is reduced, but its gravitational potential energy and spring potential energy are increased. This process continues until the highest point is reached, where its velocity and kinetic energy are zero, but its gravitational potential energy and spring potential energy are maximum. At any point during the cycle, total energy of the mass is given as follows:

(potential energy stored in spring) + (kinetic energy) + (gravitational potential energy), or —

$$1/2\ kx^2 + 1/2\ mv^2 + mgh$$

This process would continue forever if friction did not dissipate the energy gradually and reduce the vertical oscillation.

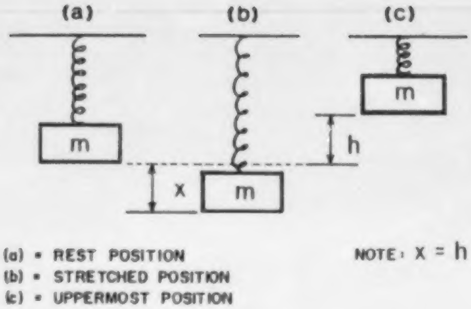

(a)
(b)
(c)
m
m
m
x
h
(a) = REST POSITION
(b) = STRETCHED POSITION
(c) = UPPERMOST POSITION
NOTE: x = h

FIG. 4

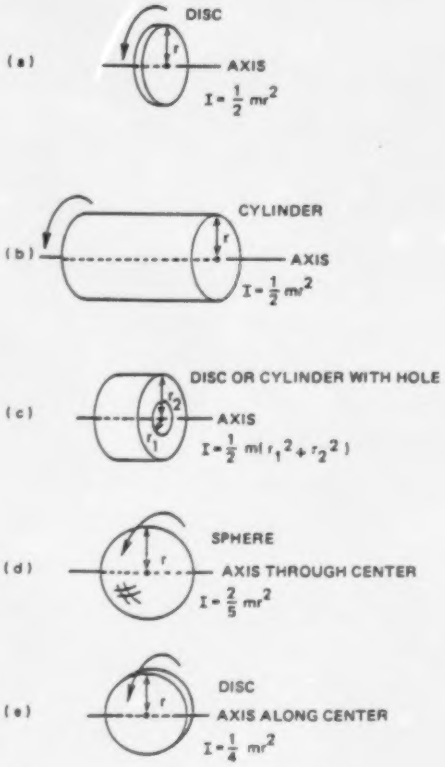

(a)
DISC
r
AXIS
I = 1/2 mr²
(b)
CYLINDER
r
AXIS
I = 1/2 mr²
(c)
DISC OR CYLINDER WITH HOLE
r2
r1
AXIS
I = 1/2 m(r1² + r2²)
(d)
SPHERE
r
AXIS THROUGH CENTER
I = 2/5 mr²
(e)
DISC
r
AXIS ALONG CENTER
I = 1/4 mr²

FIG. 5

ROTATIONAL

Concept Module 4-0, "Momentum," introduced the concept of moment of inertia and suggested that it is the quantity in a rotational system that corresponds to mass in a linear system. Recall that the moment of inertia (I) varies with the shape of the object and with the position of its axis of rotation. Formulas for determining the moment of inertia of various objects are summarized in Figure 5.

Since the moment of inertia corresponds to mass in a linear system, and since angular velocity (ω, rad/s) corresponds to the angular rate of motion, the reasonable assumption is that the formula for rotational kinetic energy is similar in form to the formula for linear kinetic energy. Thus, $1/2\ mv^2$ becomes $1/2\ I\omega^2$ when going from linear to rotational motion. Rotational kinetic energy is given in Equation 5.

$$E_K = 1/2\ I\omega^2 \qquad (5)$$

where

I = moment of inertia.

ω = angular velocity in rad/s.

Two problems that illustrate the use of Equation 5 are given in Examples C and D.

EXAMPLE C: KINETIC ENERGY OF A FLYWHEEL

Given: A 20-kg flywheel disc whose radius is 25 cm rotates about its axis with an angular velocity of 300 r/min (rpm).

Find: Its kinetic energy.

Solution:

$$E_K = 1/2\ I\omega^2$$

$$I = 1/2\ mr^2 \text{ (from Figure 5a)}$$

$$= (1/2)(20\text{ kg})(0.25\text{ m})^2$$

$$I = 0.625\text{ kg} \times \text{m}^2$$

$$E_K = 1/2\ (0.625\text{ kg} \times \text{m}^2)\left(300\ \frac{\text{r}}{\text{min}} \times \frac{\text{min}}{60\text{s}} \times \frac{2\pi\text{ rad}}{\text{r}}\right)^2$$

$$= 1/2\ (0.625)(987)\ \frac{\text{kg} \times \text{m}^2}{\text{s}^2}$$

$$E_K = 308\text{ joules}$$

EXAMPLE D: ENERGY USED TO START A LAWN MOWER

Given: A manual starting system on a lawn mower requires a force of 180 newtons to pull the cord. The homeowner pulls the rope a total of 0.8 meters when attempting to start the mower. The flywheel around which the rope is wound has a radius of 12 cm and a mass of 7 kilograms.

Find:

(a) The moment of inertial of the flywheel,
(b) work put into pulling the cord, and
(c) kinetic energy and angular velocity given the flywheel, assuming no load conditions.

Solution:

(a)

$$I = 1/2\ mr^2$$
$$= 1/2\ (7\ \text{kg})(0.12\ \text{m})^2$$
$$I = 0.05\ \text{kg} \times \text{m}^2$$

(b) Recall from Module 2-0 —

$$W = \tau\theta$$
$$\tau = F\ell$$
$$= (180\ \text{N})(0.12\ \text{m})$$
$$\tau = 21.6\ \text{Nm}$$
$$\theta = \frac{s}{r}$$
$$= \frac{0.8\ \text{m}}{0.12\ \text{m}}$$
$$\theta = 6.67\ \text{rad}$$
$$W = (21.6\ \text{Nm})(6.67\ \text{rad})$$
$$W = 144\ \text{joules}$$

or alternately —

$$W = Fx$$
$$= (180\ \text{N})(0.8\ \text{m})$$
$$= 144\ \text{Nm}$$
$$W = 144\ \text{joules}$$

(c) $E_K = \frac{1}{2} I\omega^2$

If no friction of load is present, all the work put into pulling the rope is transferred into kinetic energy of the flywheel. Thus, the kinetic energy = 144 joules.

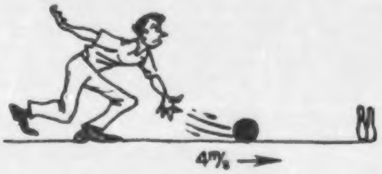
$$144\ \text{joules} = 1/2\ (0.05\ \text{kg} \times \text{m}^2)\ \omega^2$$
$$\omega^2 = \frac{2\ (144)}{0.05}\ \frac{\text{rad}^2}{\text{s}^2}$$
$$\omega^2 = 5760\ \frac{\text{rad}^2}{\text{s}^2}$$
$$\omega = 76\ \frac{\text{rad}}{\text{s}}$$

An object can be rotating at the same time that it is moving forward. If this is the case, the object has both rotational kinetic energy and translational kinetic energy. Total kinetic energy of the object would be found by determining each energy separately and then adding the results, as illustrated in Example E.

EXAMPLE E: KINETIC ENERGY OF A BOWLING BALL

Given: A 7.2-kg bowling ball with a radius of 12 cm is thrown with a velocity of 4 m/s.

Find: Total kinetic energy of the ball before it strikes the pins.



Solution: Linear kinetic energy

$E_K = 1/2\ mv^2$

$= 1/2\ (7.2\ \text{kg})(4\ \text{m/s})^2$

$E_K = 57.6$ joules

Rotational kinetic energy

$E_K = 1/2\ I\omega^2$

$I = 2/5\ mr^2$ for a sphere

$= 2/5\ (7.2\ \text{kg})(0.12\ \text{m})^2$

$= 0.04\ \text{kgm}^2$

$= 1/2\ (0.04\ \text{kgm}^2)(33.3\ \text{rad/s})^2$

$E_K = 22.2$ joules

$E_{K\text{total}} = E_{K\text{linear}} + E_{K\text{rotational}}$

$= 57.6$ joules + 22.2 joules

$E_{K\text{total}} = 79.8$ joules

Recall:

$$\omega = \frac{v}{r}$$

$$= \frac{4\ \text{m/s}}{0.12\ \text{m}}$$

$$\omega = 33.3\ \frac{\text{rad}}{\text{s}}$$

FLUIDAL SYSTEMS

Daniel Bernoulli (1700-1782) applied the conservation-of-energy principle to fluid motion. The outcome of his work is known as "Bernoulli's principle," which has wide-ranging practical applications. Devices, such as an automobile carburetor, a venturi meter, a sailboat in motion, a curving baseball, and the lift on an airplane wing, all utilize Bernoulli's principle.

Since liquids are practically incompressible, use of volume rather than mass to measure a moving liquid usually is more convenient. Therefore, kinetic and potential energy fomulas first must be modified to include volume rather than mass. This modification is accomplished, first, by recalling the definition of density ρ.

$$\rho = \frac{m}{V}$$

or,

$$m = \rho V$$

Thus, mass can be replaced in the energy equations with density times volume. The results are given in Equations 6 and 7.

$$E_K = 1/2\ (\rho V)\ v^2 \qquad (6)$$

$$E_p = (\rho V)\ gh \qquad (7)$$

where
g = acceleration of gravity (SI: 9.8 m/s^2; English: 32 ft/s^2).

Consider a fluid flowing in the pipe shown in Figure 6. Assume that the pipe has changing cross sections and does not all lie on the same level, and that the fluid flow is without resistance (zero viscosity) so that no fluid friction exists.

In a given amount of time, equal volumes of liquid will flow through section a and section b of the pipe if the fluid is incompressible.

Consider the work done in moving this volume V from section a to section b. The pressure drop through which the fluid moves is $p_a - p_b$. Since the fluid in section a is moved through the volume V, the work done on the fluid is given by Equation 8.

$$W = (p_a - p_b)\ V \qquad (8)$$

where
p_a = pressure at section a.
p_b = pressure at section b.

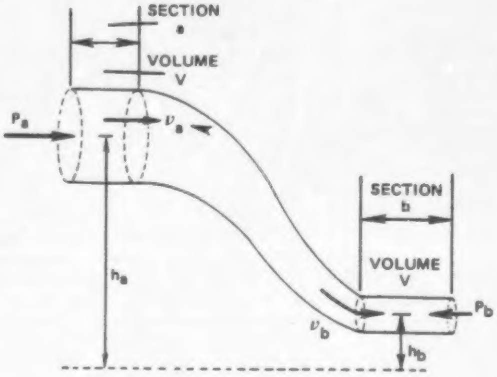


FIG. 6

Change in the E_K of the volume V of fluid is given by Equation 9.

$$\Delta E_K = 1/2\ (\rho V)\ v_b^2 - 1/2\ (\rho V)\ v_a^2 \qquad (9)$$

where
v_a = fluid velocity at section a.
v_b = fluid velocity at section b.

Change in the gravitational potential energy of the volume V of fluid is given by Equation 10.

$$\Delta E_P = (\rho V)\ gh_b - (\rho V)\ gh_a \qquad (10)$$

where
h_a = height of section a.
h_b = height of section b.

If energy is to be conserved, the work done can show up only as a change in kinetic and potential energies of the volume of fluid. Thus —

$$W = \Delta E_K + \Delta E_P$$

$$(p_a - p_b)\ V = 1/2\ (\rho V)\ v_b^2 - 1/2\ (\rho V)\ v_a^2 + (\rho V)\ gh_b - (\rho V)\ gh_a$$

This arrangement is divided through by V and the terms rearranged to yield Equation 11.

$$p_a + 1/2\ \rho v_a^2 + \rho gh_a = p_b + 1/2\ \rho v_b^2 + \rho gh_b \quad (11)$$

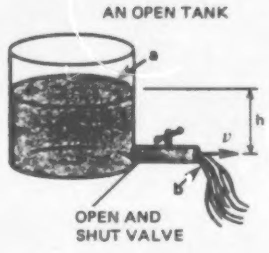


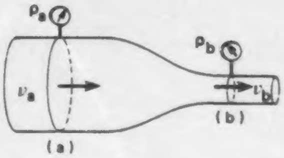
FIG. 7

Equation 11 is Bernoulli's equation and, as pointed out earlier, is a direct result of the conservation of energy. It states that the quantity (p + 1/2 ρv^2 + ρgh) must be the same at any cross section of the flow pipe. If the flow is horizontal, then Bernoulli's principle leads to the "rule of thumb" that "when the velocity of a moving fluid is high, its pressure difference is low, and when the velocity is low, the pressure difference is high."

Example F given an application of Bernoulli's equation.

EXAMPLE F: MEASURING PRESSURE CHANGES IN A PIPE WITH VARYING DIAMETERS

Given: Water flows through the horizontal pipe shown, with a velocity at "a" of 4 m/s and at "b" of 10 m/s. The pressure gage at point "a" reads 250×10^3 N/m^2.



Find: The pressure-gage reading at point "b".

Solution: Since the center of the pipe stays at the same elevation, potential energy terms in Equation 11 are equal, and Equation 11 reduces to the following:

$$p_a + 1/2\ \rho v_a^2 = p_b + 1/2\ \rho v_b^2$$

Substitution of the proper values into the above equation yields —

(Recall that ρ for water equals 1 g/cm^3 or 1000 kg/m^3.)

$$250 \times 10^3\ \frac{N}{m^2} + 1/2\ (1000\ \frac{kg}{m^3})(4\ \frac{m}{s})^2 = p_b + 1/2\ (1000\ \frac{kg}{m^3})(10\ \frac{m}{s})^2$$

$$[250 \times 10^3 + 8 \times 10^3]\ \frac{N}{m^2} = p_b + 50 \times 10^3\ \frac{N}{m^2}$$

$$[258 \times 10^3 - 50 \times 10^3]\ \frac{N}{m^3} = p_b$$

$$p_b = 208 \times 10^3 \frac{N}{m^2}\quad \text{(newtons/square meter)}$$

Consider a tank filled with fluid (Figure 7).

When the valve at the bottom of the tank is opened, fluid escapes with velocity v. Bernoulli's equation can be applied to the cross section at the upper surface of the reservoir and at the point of outflow.

At the upper surface, pressure is atmospheric pressure p_a, and velocity is negligible, $v_a = 0$. The left side of Equation 11 is then —

$$p_a + \rho g h$$

At the exit pipe, pressure again is atmospheric pressure. The height of fluid above this point is $h = 0$, and the velocity is that of the escaping fluid. Thus, the right side of Equation 11 is as follows:

$$p_b + 1/2\ \rho v^2$$

Bernoulli's principle requires the left- and right-hand quantities be equal:

$$\cancel{p_a} + \rho gh = \cancel{p_b} + 1/2\ \rho v^2 \qquad (12)$$

Since $p_a = p_b$ = atmospheric pressure —

$$\rho gh = 1/2\ \rho v^2$$

Solving for v —

$$v^2 = 2\ gh$$

or

$$v = \sqrt{2\ gh} \qquad (13)$$

The conservation-of-energy principle has allowed determination of the exit velocity of a fluid from a tank system. This is illustrated in Example G.

EXAMPLE G: CALCULATING EFFLUX VELOCITY

Given: A tank filled with 15 feet of water is equipped with a valve at the bottom, as shown in Figure 7.

Find: (a) Velocity of efflux. (b) If the spigot is aimed upward, how high will the water rise? (c) If the water actually rises less than the amount predicted, how can this be accounted for?

Solution: (a)

$$\begin{aligned} v &= \sqrt{2\ gh} \\ &= \sqrt{2\ (32\ \text{ft/s}^2)(15\ \text{ft})} \\ &= \sqrt{960} \\ v &= 31\ \text{ft/s} \end{aligned}$$

(b) If the spigot were pointed upward, kinetic energy of the water would be turned back into potential energy, and the water should rise to the original 15-foot height.

(c) If the water does not actually rise 15 feet, the discrepancy can be explained in the following way:

Friction within the fluid and of fluid moving against walls of the pipe and valve has consumed a portion of the energy.

ELECTRICAL SYSTEMS

Consider now the concept of potential energy in electrical-energy systems as illustrated by the electrical circuit in Figure 8. The circuit consists of a capacitor connected to a battery.

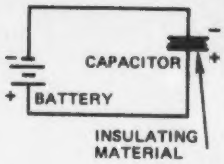


FIG. 8

A capacitor consists of two conducting plates separated by an insulating material. The battery in Figure 8 provides a potential difference V that forces electrons toward the capacitor. The charge that reaches the plate of the capacitor cannot move through the insulating material to reach the other plate. Opposite and equal charge builds up on the capacitor plates according to the plus (+) and minus (-) polarity shown in Figure 8. A stage will be reached when the battery cannot force any more charge onto the capacitor plates.

The quantity of charge that builds up on either plate when the potential difference is one volt is called "capacitance." Capacitance depends upon area of the capacitor plates, distance between the plates, and the type of insulating material between them. Capacitance is said to be one farad when a one-volt battery deposits a charge of one coulomb on each capacitor plate. The definition of capacitance is given mathematically in Equation 15.

$$C = \frac{Q}{V} \tag{15}$$

where

C = capacitance.
Q = charge stored.
V = voltage.

The farad is a very large unit of capacitance; therefore, capacitors frequently are rated in microfarads ($\mu F = 10^{-6}$ F) or micro-microfarads ($\mu\mu F = 10^{-12}$ F). A micro-microfarad is also referred to as a "picofarad" ($pF = 10^{-12}$ F).

When a capacitor is charged in the manner described earlier, the charge remains on the capacitor for a time, even after the battery is removed. This stored charge represents a kind of potential energy. Anyone who accidentally touched a charged capacitor can verify the fact that energy is stored in the device! Energy stored by a capacitor is very similar, in principle, to the energy stored in a stretched spring.

Equation 16 provides a way to determine the amount of energy stored in a capacitor.

$$E_p = 1/2\ QV \qquad (16)$$

From Equation 15, $Q = CV$; and if this is substituded into Equation 16, two more additional useful forms for potential energy result:

$$E_p = 1/2\ \frac{Q^2}{C} \qquad (17)$$

$$E_p = 1/2\ CV^2$$

Example H illustrates the use of Equations 15 and 17.

EXAMPLE H: FUNCTION OF A CAPACITOR

Given: Twelve volts are applied to charge a 0.5-μF capacitor.

Find:
(a) Amount of charge stored on the plates.
(b) Amount of potential energy stored by the capacitor.
(c) What becomes of the energy if the plates are shorted together.

Solution: (a) $C = \frac{Q}{V}$

or

$$Q = CV$$
$$= (0.5 \times 10^{-6}\ F)(12\ V)$$
$$Q = 6 \times 10^{-6} \text{ coulombs}$$

(b) $E_p = 1/2\ CV^2$

$$= 1/2\ (0.5 \times 10^{-6}\ F)(12\ V)^2$$
$$E_p = 3.6 \times 10^{-5} \text{ joules}$$

(c) When the capacitor is shorted, a current flows in the connecting wire until the charges are neutralized. This current produces heat energy. A spark probably will occur. Some energy is given off as light when a spark occurs. Thus, most of the energy is dissipated as heat energy, and some is given off as light energy.

HEAT SYSTEMS

Up to this point, the discussion of kinetic energy has been easy to visualize in a moving object. Heat energy is not as easily "seen" as other forms of energy.

However, kinetic energy is involved in heat energy, because heat is nothing more than the sum total of all the kinetic energy possessed by the atoms of the object. The temperature of a substance is a measure of the average kinetic energy of its atoms.

When heat is transferred to or from an object, there is a corresponding change of temperature. The amount of transferred heat energy can be determined with the help of Equation 18.

$$H = mc\Delta T \qquad (18)$$

where

H = thermal energy added to a body to cause a temperature change ΔT.

ΔT = temperature change of the body.

m = mass of the body.

c = constant characteristic of the body, known as the "specific heat" of the substance.

Recall that, by definition, water requires 1 calorie of heat energy to raise the temperature of 1 gram by 1 degree Celsius. Specific heat of water is then 1 cal/g x °C. A specific heat such as 0.22 for aluminum means that only 0.22 a calorie is needed to change the temperature of one gram of aluminum one degree Celsius. Specific heats for other materials easily are found in standard reference handbooks.

Heat energy (molecular kinetic energy) can be transferred by three different ways: conduction, convection and radiation.

When two objects at different temperatures are put in contact, the rapidly-moving molecules of the hot object collide with, and transfer energy to, the molecules of the cold object. This is the basic process of heat conduction. When liquids and gases are heated, the hot material expands, causing a reduced density. The lighter, hot material floats upward, causing convection currents. In this way, heat energy is transported by convection. All objects that have a temperature above absolute zero emit heat energy in the form of electromagnetic radiation, and radiation is the only process by which heat energy can be transmitted through a vacuum. This phenomenon will be discussed in greater detail in Concept Module 13-0, "Radiation."

SUMMARY

In this discussion of energy, there is a striking similarity between certain mathematical representations of energy, which reveals analogies between physical systems. For example, the similarity of kinetic energy $1/2\ mv^2$ in the translational and $1/2\ I\omega^2$ in the rotational mechanical systems reveals the analogous roles in the two system played by v and ω (ratelike quantities) and by m and I (mass or inertialike quantities.) Also, the similarity of potential energies $1/2\ kx^2$ and $1/2\ (1/C)Q^2$ in the mechanical and electrical systems reveals the analogous roles played by x and Q (displacementlike quantities) and by k and $1/C$.

Energy itself is a fundamental unifying concept, because it is common to all systems. In all systems we notice that whenever work is done, there is an increase in kinetic or in potential energy, or in both. When energy is transformed from one system to another, we expect the total amount of energy present to remain the same, including heat energy produced by friction. If friction is present, a portion of the work is represented by a frictional heat energy loss. In the absence of friction, the sum total of the kinetic and potential energy reamins constant, although there may be exchanges between these two forms of energy. This fact, known as the "conservation of energy," is one of the fundamental principles upon which all science and technology is based.

MECHANICAL

MATERIALS

Inclined track.
Metal sphere, cylinder and ring or other similar objects.
Meter stick.
Stopwatch.

PROCEDURE

The principle of the conservation of energy will be applied to several objects rolling down an incline. Potential energy of the objects will be measured at the top of the incline, and translational and rotational kinetic energy will be calculated at the bottom of the incline.

1. Arrange a ball, cylinder, or ring at top of an inclined track. Measure height of object; calculate its potential energy from equation $E_p = mgh$.

   mass = ____________
   g = ____________
   h = ____________
   Ep = ____________

2. As object rolls, it will lose potential energy and gain kinetic energy (both rotational and translational) according to the principle of conservation of energy:

$$mgh = 1/2\ I\omega^2 + 1/2\ mV^2.$$

3. Measure v at bottom of incline. To accomplish this measurement, allow object to roll on a friction-free level surface after it comes off incline. Record time t required for object to travel a premarked, precisely-measured, level distance d; calculate v from the following equation:

$$v = \frac{d}{t}$$

4. Calcualte and record translational kinetic energy of object.

   m = ____________

$v =$ ________

$E_K = 1/2\ mv^2 =$ ________

5. Calculate angular velocity ω from linear velocity v, using this equation:

$$\omega = \frac{v}{r}$$

Here, r is radius of the rolling object. Calculate rotational kinetic energy, $1/2\ I\omega^2$, using a value for I from Figure 5 for shape of the object used. (A ball would be easiest.)

$r =$ ________

$\omega = \frac{v}{r} =$ ________

$I =$ ________

$E_K = 1/2\ I\omega^2 =$ ________

6. Compare total kinetic energies as found in Steps 4 and 5 with original potential energy as found in Step 1. Is energy conserved, as stated in Step 2? Calculate a % difference.

$$\%\ \text{difference} = \frac{E_P - (E_K)\ \text{total}}{E_P} \times 100$$

7. Repeat entire procedure with other shaped objects as time permits.

FLUIDS

MATERIALS

Pail with hole in side near bottom.
Vernier caliper.
2-liter beaker or other volume measure.

PROCEDURE

The exit velocity of a fluid will be obtained in two different ways and the two values compared.

1. Obtain a pail with a small, drilled, circular hole in side near bottom. Fill paid to a line near top. Allow water to discharge from pail without restriction. Adjust water entering pail in such a manner that water level stays at same elevation throughout experiment (Figure 9).

2. Measure height h of water in pail, and calculate exit velocity from $v = \sqrt{2gh}$, as described in text.

   h = ________

   v = ________

3. To verify that this equation is valid, also obtain v in the following way: Recall from earlier modules that volume flow rate of a fluid $\Delta V/\Delta t$ is equal to velocity v of the fluid times the area A through which the fluid is moving.

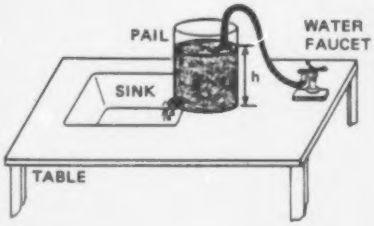


FIG. 9

$$\frac{\Delta V}{\Delta t} = Av$$

4. Record time necessary to fill a 2-liter beaker or other known volume with water coming from the hole. Calculate flow rate in cm^3/s. ________________

5. Using a vernier caliper or other precision measuring tool, determine drilled diameter of hole and calculate its area in cm^2.

6. With equation given in step 3 above, calculate velocity of discharged fluid.

   $\Delta V/\Delta t$ = ________ cm^3/s

   A = ________ cm^2

   v = ________ cm/s

7. Compare the two values obtained for the exit velocity by finding percentage variation?

$$\% \text{ variation} = \frac{\text{difference}}{\text{average}}$$

ELECTRICAL

MATERIALS

Small hand-operated d.c. generator.
Capacitor 10 µF, 200 volt.
Voltmeter.
Ammeter.
Spring scale.

PROCEDURE

The following values will be determined: (1) mechanical energy necessary to rotate a small electric generator used to charge a capacitor, and (2) efficiency of the system.

1. A hand-operated d.c. generator is necessary for this experiment. If only an a.c. generator is available, place a diode or rectifier in circuit to change a.c. to d.c.

2. Connect circuit as shown in Figure 10.

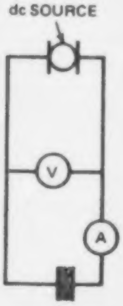


FIG. 10

3. Calculate mechanical energy input from the work equation W = Fx. It will be necessary to use a spring scale on end of crank to determine force necessary to turn generator.

   F = __________ newtons

   Turn generator uniformly (counting revolutions) until a full charge is obtained on capacitor. When a full charge is obtained, no current will flow through ammeter. From number of revolutions N and radius of crank r, determine distance x moved to obtain a full charge.

   $$x = 2\pi r N \text{ (meters)}$$

   Work input = Fx

   Work input = ________ joules

4. Calculate potential energy stored in capacitor from equation $E_p = 1/2\ CV^2$, as described in text.

   Ep = __________ joules

5. Considering energy stored in capacitor as the work output, calculate efficiency of this system:

   $$\text{efficiency} = \frac{\text{work output}}{\text{work input}}$$

   efficiency = ___________

SELF CHECK

1. Define potential energy, kinetic energy, and conservation of energy, using examples from mechanical systems.

2. An automobile weighing 2800 lb is traveling at 55 miles per hour. What is its kinetic energy?

3. What would the speed of the automobile in problem 2 be if its kinetic energy were doubled?

4. An 1800-lb elevator is lifted a vertical distance of 70 ft by an electric motor. Calculate potential energy of the elevator at the 70-ft height.

5. A spring requires a force of 25 N to stretch it 15 cm. What is the spring constant (k)? How much energy is stored in the spring when it is stretched 45 cm?

6. An electrical capacitor of 1.5 μF is connected to a 6.0-V battery. How much potential energy is stored in the capacitor?

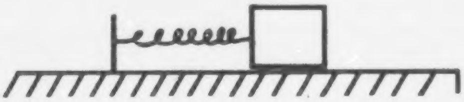
7. A solid wheel (disk) with a mass of 5 kg has a speed of 20 m/s. Determine its total kinetic energy.

8. Starting with Bernoulli's equation, derive the expression for the velocity of a frictionless fluid flowing from an opening in a large tank; and use this equation to determine the velocity of water exiting through a hole 50 ft below the water level of a tank.

9. Explain similarities and differences between conduction, convection, and radiation.

10. Discuss conservation of energy as it applies to mechanical, fluidal, electrical, and thermal systems.

11. A 1.0 kg mass is fixed to the end of a spring which is on a horizontal, frictionless table as shwon in the figure below. If the spring is stretched 10 cm beyond its rest point and then released how fast will the mass be moving just as it passes through the rest point? Assume that the spring has a spring constant of 100 N/m and that there are no sources of dissipative friction.



12. How high is a waterfall if 3000 kg of water acquires a kinetic energy of 400,000 joules by falling from the top of the waterfall to the ground?

13. A 40 kg mass rests on one end of a teeter-totter. A 300 kg mass is dropped from a height of 10 meters. If 40% of the

larger block's energy is transferred to the small mass what is the height to which the small mass rises?

14. What capacitance should a capacitor have if it is to store 3×10^{-3} joules of energy when connected to a 12-volt battery?

15. A 400 kg mass is dropped from a height of 40 meters. Roughly sketch the block's potential energy versus time and the block's kinetic energy versus time on the same graph.

16. A solid wheel's total kinetic energy is 3000 joules. What is the wheel's spped if its mass is 7 kg?

17. What is the ratio of a translational to rotational kinetic energy of a sold wheel moving at a constant translational speed?

18. How much charge is stored on the plates of a parallel plate capacitor if the capacitor stores 5×10^{-4} joules of energy when the applied voltage is 24 volts?

19. What capacitance should be placed in parallel with a 10-μF capacitor if the applied voltage is to be 10 V when the system contains 10^{-3} J of potential energy? (Hint: capacitance in parallel, add.)

20. How high above a valve at the bottom of a tank is the water level in the tank when 10 m^3 of water flows out of the tank per minute if the area of the valve opening is 0.1 m^2?

21. How much heat must be added to a 1-kg block of aluminum to increase the temperature of the block by 20C°?

22. How much water at 40°C could be brought to 100°C using energy produced by a rotating 5-kg flywheel that has a radius of 50 cm and an angular velocity of 20 rad/sec if all of the rotational energy of the flywheel went into heating the water?

23. What is the speed of a 0.1-kg ball dropped from a height of 20 meters at the instant before the ball hits the ground?

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FORCE TRANSFORMERS

CHAPTER 8-0

There are times when nature seems to give something for nothing. However, closer examination when comparing some output to some input shows that nature insists on the principles of conservation of energy. More energy cannot be taken out of a system than put into the system.

This module treats a class of devices called "force transformers" because, in such devices, the force output is different from the force input. However, the work, or energy output, is never greater than the work, or energy, input; and with an ideal case in which no energy is lost to frictional forces, the work output equals the work input.

Force transformers are simply machines having great diversity and having a wide variety of applications, but the basic explanation of their usefulness is the same. The unified approach taken in the preceding modules will assist in understanding force transformers in different energy systems as examples of the same physical principle.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Describe specific force transformers in the mechanical translational, mechanical rotational, fluidal, and electrical systems; and discuss their fundamental similarity as transformers of forcelike quantities.

- Define the following terms:

  Ideal mechanical advantage.
  Actual mechanical advantage.
  Efficiency.

- Calculate the ideal mechanical advantage of a specific pulley, lever, screw, wheel and axle, hydraulic press or lift, and electrical transformer.

- Calculate the change in current in an ideal electrical transformer.

- Discuss how the role of resistance in a transformer dissipates energy input and reduces efficiency.

- Describe the power input and power output characteristics of a transformer that operates continuously.

PRINCIPLES OF FORCE TRANSFORMERS

The prospect of changing a flat tire without a lever jack would be a dismal one. With a jack, however, the relatively weak muscular forces of the human body are transformed, or magnified, sufficiently to raise one end of a two-ton car off the ground. Levers and screw jacks are examples of force transformers in mechanical systems.

An example of force transformers in fluid systems is a barber's chair. When the barber wishes to elevate the customer to a more convenient height, the barber makes use of a hydraulic pump. With a few effortless motions of a lever, the client is lifted in the chair.

Consider, finally, force transformers in electrical systems. Good examples of these devices are electric calculators and cassette recorders that operate from a 110-volt line. Ordinarily, these devices operate from batteries, which typically provide 6 volts. The accomodation is made by "stepping down" the 110 volts to the required 6 volts by use of a "6-volt transformer." Here, voltage, rather than mechanical force, is being transformed; but recall that in the electrical system, voltage is the "forcelike quantity." Thus, the lever/screw jack and the 6-volt transformer appear to be similar physical systems.

In each of the examples considered above there is a coupling device, or transformer, that receives an input from some source and produces an output fed to some load, as shown schematically in Figure 1.

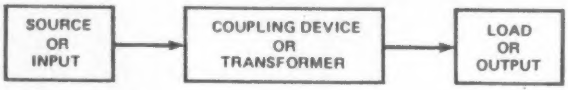


FIG. 1

Re-examination of the three examples of force transformers given above according to this scheme shows that in the first example, the source is an individual operating the jack. The individual's input is the muscular force exerted by their arm. The coupling device is the screw jack. The load is the car, and the output is the force required to lift one end of the car off the ground. (These facts are summarized in Figure 2.)

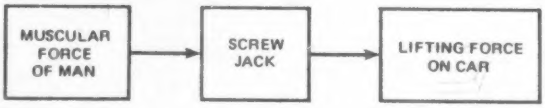


FIG. 2

In this case, the function of the screw jack is to multiply the input force sufficiently to lift the car with the required output force.

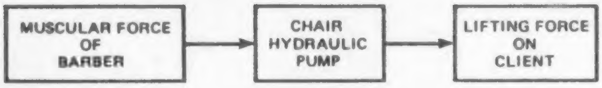


FIG. 3

The other examples can be diagrammed in the same way (Figures 3 and 4).

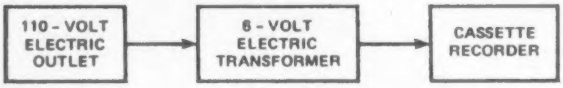


FIG. 4

Energy is produced by work done at the source, and is consumed as work done on the load. Recall the following from Module 2-0:

$$\boxed{\text{WORK} = \text{FORCELIKE QUANTITY X DISPLACEMENTLIKE QUANTITY}} \qquad (1)$$

The coupling device, or force transformer, transfers the energy from the source to the load. Although the forcelike quantities and the displacementlike quantities at each end may have different magnitudes, the product of these quantities (which is the energy transferred from input to output) is ideally the same. In practice, the output energy will be somewhat less, because some of the energy will be dissipated as heat from friction in the operation of the transformer itself. Energy is conserved overall, but some is wasted in overcoming resistance. Naturally, one tries to reduce these losses in order to make a highly-efficient transformer. The percent efficiency of a transformer is defined by Equation 2.

$$\eta = \frac{E_{out}}{E_{in}} (100) \qquad (2)$$

where

η = % efficiency.
E_{out} = useful output energy.
E_{in} = input energy.
100 = multiplier to obtain %.

When useful output energy equals input energy (the ideal case), no energy is lost in the coupling device; and the formula gives 100% efficiency. In actual cases, the output energy is somewhat less, and the formula gives an efficiency of less than 100%. In no case can the efficiency be greater than 100%.

In the discussion that follows, ideal, or 100%, efficiency will be assumed, unless otherwise specified. Keep in mind that real transformers will be less efficient, because of energy losses in the actual operation. The actual efficiency cannot be predicted in advance, because it will depend on how successively the device can be made to operate without frictional affects.

MECHANICAL FORCE TRANSFORMERS

THE PULLEY

A simple pulley is shown in Figure 5. A weight W is lifted by pulling on the cord with a force F. Experimentation with this system shows that the required force F is equal to the weight W if there is no friction in the pulley; otherwise, F must be greater than W. This arrangement does not multiply force; it is not, therefore, a force transformer. About all that can be said of this device is that it changes the direction of the force. When the rope is pulled downward, the weight is lifted upward. The device may be useful, but it is not a force transformer. The pulley pictured in Figure 6 is, however, a force transformer.

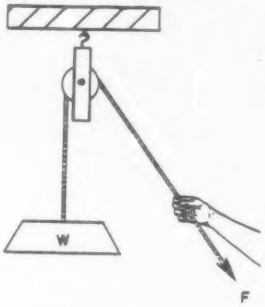


FIG. 5

Study the figure, and notice that if the rope is pulled (or F_2 moves) a distance of 1 foot, the load will be raised (or F_1 moves) only 1/2 foot. Thus, x_2 will always be twice x_1, or $x_1/x_2 = 1/2$. When the arrangement is used for an ideal transformer, input work = output work, as stated by Equation 3.

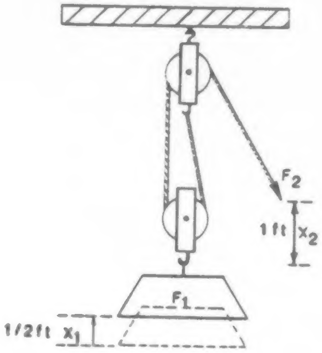


FIG. 6

$$F_2x_2 = F_1x_1$$

or

$$F_2 = (x_1/x_2)\ F_1 \qquad (3)$$

where

F_2 = input force at the source.
F_1 = output force at the load = weight to be lifted.
x_2 = vertical distance moved by input force.
x_1 = vertical distance moved by output force or weight.

The necessary conclusion is that $F_2 = 1/2\ F_1$ and that the weight is lifted by pulling with a force of only 1/2 the weight. This pulley arrangement definitely is a force transformer. One "pays" for the force advantage by being required to pull twice as far as the weight actually rises. Notice that this arrangement makes the product of force and displacement the same at the source and load ends. Study this argument carefully, because it is the basic argument that applies to all ideal force transformers: What one gains as a force advantage, one gives up as a displacement disadvantage; but the product of force and displacement, representing work, must be the same at input and output.

Because a load twice as large as the applied force can be lifted with the pulley in Figure 6, this pulley "has a mechanical advantage of 2." In general, for an ideal transformer, mechanical advantage is given by Equation 4. Equation 4 is used to solve problems in Examples A and C.

$$M_i = \frac{x_2}{x_1} \tag{4}$$

where
M_i = ideal mechanical advantage.

From Equation 3, notice that the ratio of x_2/x_1 is identical with the ratio F_1/F_2 for an ideal transformer. Clearly, then, the ratio F_1/F_2 could be used as well as x_2/x_1 to determine mechanical advantage. In an actual pulley, since there will be some friction involved, the input force will have to be slightly larger to do the job. The mechanical advantage F_1/F_2 in practice then would have to be determined by measurements of both F_2 and F_1 in an actual test of the pulley. For an actual transformer, the mechanical advantage is given by Equation 5, used to solve problems in Examples A, B, and C.

$$M_a = \frac{F_1}{F_2} \tag{5}$$

where
M_a = actual mechanical advantage.

If the transformer is frictionless, either formula will give the same result for mechanical advantage.

A little algebra will show that the formula for % efficiency (Equation 2) is equivalent to various other convenient forms. Any of the following forms given in Equation 6 can be used as well:

$$\eta = \frac{M_a}{M_i}(100) \tag{6}$$

or

$$\eta = \frac{F_2(\text{ideal})}{F_2(\text{actual})}(100)$$

or

$$\eta = \frac{F_1\,(\text{actual})}{F_1\,(\text{ideal})}\,(100)$$

EXAMPLE A: THE PULLEY AS A FORCE TRANSFORMER

Given: In the pulley system shown in Figure 6, a load of 400 lb is lifted 5 ft. (Assume that this system is an ideal transformer.)

Find: (a) The amount of work that is done by the source, (b) the applied force F_2, and (c) distance that the input force moves.

Solution: In this case, the pulley has an ideal mechanical advantage of 2.

(a) Output work:

$$\begin{aligned} W_{out} &= F_1 x_1 \\ &= (400)(5) \\ W_{out} &= 2{,}000\ \text{ft}\cdot\text{lb} \end{aligned}$$

(b) Applied force:

$$\begin{aligned} F_2 &= \frac{F_1}{M} \\ &= \frac{400}{2} \\ F_2 &= 200\ \text{lb} \end{aligned}$$

(c) Since $M_i = x_2/x_1$, the input force displacement corresponding to the load displacement of 5 ft can be found from:

$$x_2 = (M_i)\ x_1 = (2)(5\ \text{ft}) = 10\ \text{ft}$$

EXAMPLE B: MECHANICAL ADVANTAGE AND EFFICIENCY OF PULLEY

Given: When actually measured, the applied force in Example A is found to be 225 lb.

Find: (a) The actual M of the pulley and (b) its % efficiency.

Solution: (a)

$$\begin{aligned} M_a &= \frac{F_1}{F_2} \\ &= \frac{400}{225} \\ M_a &= 1.78 \end{aligned}$$

(b) $\eta = \frac{M_a}{M_i}(100)$ (from Equation 6)

$= (1.78/2)(100)$

$\eta = 89\%$

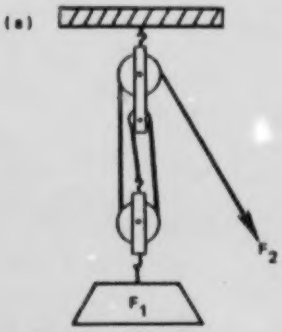
EXAMPLE C: MECHANICAL ADVANTAGE OF BLOCK AND TACKLE

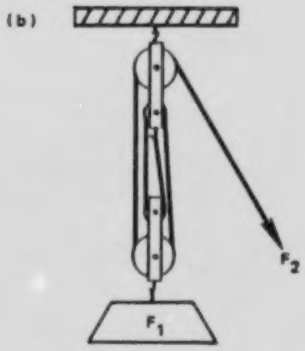
Given: Pulley systems a and b as shown. A combination of pulleys of this type is called a "block and tackle."

Find: The ideal mechanical advantage of the pulleys shown.

Solution: (a) F_1 must move down vertically three times as much as F_1 moves up, if the cord is not to stretch. Thus, $M_i = x_2/x_1 = 3/1 = 3$

(b) In this case, by similar reasoning, $M_i = x_2/x_1 = 4/1 = 4$. Notice that the ideal M_i turns out to be just the number of lengths or cord supporting the load, not counting the one being pulled on directly.





THE LEVER

The lever, essentially, is a rigid bar that can be pivoted at some point. Consider the crowbar (Figure 7a) used to move a heavy rock. A smaller stone serves as the pivot point. The input force F_2 is transformed to an output force F_1 large enough to move the boulder. Figure 7b illustrates the lever with dimensions before and after the boulder is raised. The dotted line indicates the final position of the lever. Notice that the applied force F_2 moves through a larger distance x_2 than the load, which is lifted a distance x_1. The force advantage is gained, as with the pulley, at the expense of displacement.

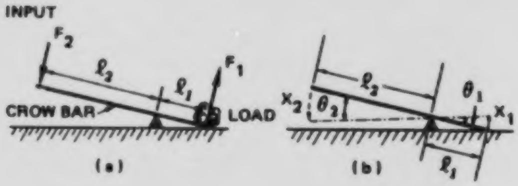


FIG. 7

The mechanical advantage of the ideal lever can be found from the formula arrived at in Equation 4. When we refer to Figure 7, we conclude that $M_i = x_2/x_1$. This equation can be put into a more useful form by relating the displacements to the lever arms of the lever; that is, the distance from each end to the pivot point. Notice that angles θ_2 and θ_1 are equal, so that —

$$\frac{x_2}{x_1} = \frac{\ell_2}{\ell_1}.$$

Since the two ratios are the same, one can write Equation 7 as follows:

$$M_i = \frac{\ell_2}{\ell_1} \qquad (7)$$

where

ℓ_1 = lever arm of load force.
ℓ_2 = lever arm of applied force.

According to Equation 7, ℓ_2 must be greater than ℓ_1 to magnify the force input. The greater force advantage achieved, the larger the ratio of ℓ_2 to ℓ_1, or the nearer the load to the pivot point. (See Examples D and E, where Equation 7 is used to solve problems.

EXAMPLE D: MECHANICAL ADVANTAGE OF A LEVER

Given: The boulder of Figure 7 weighs 500 lb, and the input force on the 6'0" crowbar cannot be greater than 100 lb.

Find: The point at which the pivot stone should be placed to move the boulder.

Solution: The lever must transform an input force of 100 lb to an output force of 500 lb; therefore, the required mechanical advantage is 500/100 = 5. Then —

$$M_i = 5 = \frac{\ell_2}{\ell_1}$$

or

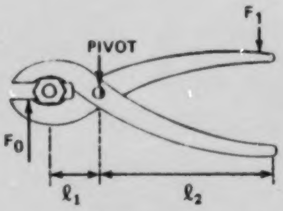
$$\ell_2 = 5\,\ell_1$$

Since $\ell_2 + \ell_1 = 6$ ft (the length of the crowbar), the algebraic solution of the last two equations must be $\ell_2 = 5$ ft, and $\ell_1 = 1$ ft. Thus, the pivot must be no further than 1 ft from the load end of the lever.

EXAMPLE E: PLIERS AS A LEVER

Given: A pair of pliers.

Find: The reason why a pair of pliers is effective in removing a stubborn nut.



Solution: The pliers actually is a two-bar lever. If one of the handles (say, the lower one) is considered fixed, then the mechanical advantage is as follows:

$$M_i = \frac{\ell_2}{\ell_1} > 1$$

THE INCLINED PLANE

A crate is moved more easily into the back end of a truck by use of an incline than by a direct vertical lift (Figure 8). The inclined plane serves as another coupling device that enables a small force to do a big job at the expense of a force displacement. In this case, the crate is rolled along the incline further than it is lifted; that is, $x_2 > x_1$. This "sacrifice" in displacement shows up as a gain in force, or a mechanical advantage. Work done by the source and in lifting the load is indicated thusly:

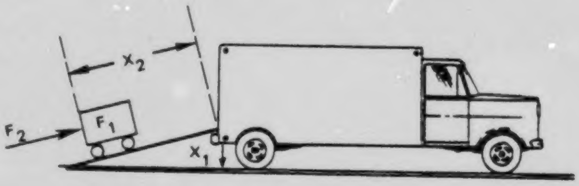


FIG. 8

$$W_2 = F_2x_2$$
$$W_1 = F_1x_1$$

where

W_2 = work done by source.
W_1 = work done in lifting the load.

If the rollers are well-greased in such a manner that the process is practically friction free, the input work is nearly equal to the output work, or —

$$F_2x_2 = F_1x_1.$$

Then the load force $F_1 = (x_2/x_1)\ F_2$. The mechanical advantage giving a force magnification of F_1/F_2 can be written for an inclined plane in the form of Equation 8.

$$M_i = \frac{x_2}{x_1} \tag{8}$$

where

x_1 = rise of incline.
x_2 = length of incline.

Equation 8 is employed in Example F to solve a problem.

Notice that this equation is just another form of Equation 4. The ratio of incline to rise sometimes is called the "grade" of the incline, so that the M_i of an incline plane is easily remembered as simply the grade of the incline. (See Examples F and G.) This ideal M_i would not be achieved if it were necessary to slide (rather than to roll) the crate up the incline. Why?

EXAMPLE F: MECHANICAL ADVANTAGE OF AN INCLINED PLANE

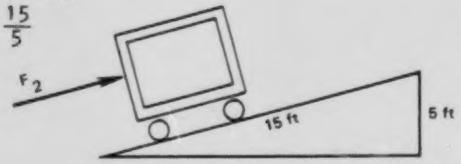
Given: A 300-lb safe is pushed up an incline 15 ft long and 5 ft high. (Assume that the rollers are frictionless.)

Find: (a) The grade of the incline, (b) the M_i of the incline, and (c) the force F_2 required to roll the safe up the incline.

Solution: (a) $\text{Grade} = \dfrac{\text{inclined length}}{\text{rise}}$

$= \dfrac{x_2}{x_1} = \dfrac{15}{5}$

$\text{Grade} = 3$



(b) $M_i = \text{grade}$

$M_i = 3$

(c) $F_2 = \dfrac{F_1}{F_2}$

$= \dfrac{15}{5}$

$F_2 = 3$

or

$F_2 = \dfrac{F_1}{e}$

$= \dfrac{300}{3}$

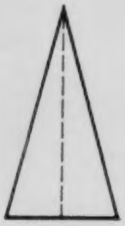
$F_2 = 100 \text{ lb}$

EXAMPLE G: THE WEDGE AS A AN INCLINED PLANE

Given: A "wedge" used to split logs.

Find: Why the wedge makes the job easier.

Solution: The wedge is one variation, or application, of the inclined plane. Notice that the wedge is just two inclined planes joined together back to back. Its M_i allows the use of a driving force which actually is smaller than the sideways force required to split the log.



THE SCREW

The screw is a clever adaptation of the inclined plane in the rotational mechanical system. The ordinary wood screw (Figure 9), for example, really is an inclined plane wrapped into a spiral around the screw axis.

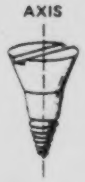


FIG. 9

In order to appreciate the transforming advantage of the screw, consider a type screw jack used to raise the corner of a house (Figure 10). Rotation of the lever forces the loading platform on the screw upwards, lifting the load. Notice that this device combines two force transformers — the lever and the screw. When the handle is turned by an input force F_2 through the distance of one revolution, the load rises by a much smaller distance, equal to the linear displacement by one thread of the screw. This distance between threads of the screw usually is called the screw "pitch." If the screw has 5 threads per linear inch, for example, the load will be lifted 1/5 inch for each revolution of the screw.

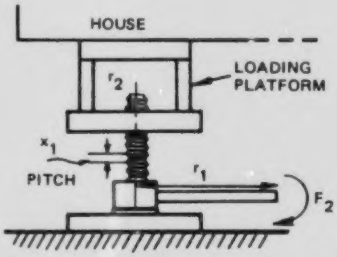


FIG. 10

Equation 4 is used again for ideal M_i. Now the relative displacement of the force F_2 and of the load F_1 must be determined. For one revolution of the handle, the force moves through one circumference of a circle with radius r_2, as shown. The corresponding displacement of the load is equal to the pitch of the screw. Therefore, Equation 9 can be written thusly for the ideal M_i of a screw.

$$M_i = \frac{2\pi r_2}{x_1} \tag{9}$$

where

$2\pi r_2$ = circumference of input force of circle.

π = pi, 3.14.

x_1 = pitch of screw (distance between threads).

In such applications of the screw, the load usually is so large that frictional forces of turning are appreciable, even when the screw is well-lubricated. As a result, the actual M_i is considerably less than that predicted by Equation 9. As explained previously, the actual M_a can only be determined by measuring the source force F_2 needed to move the load F_1. Then M_a(actual) = F_1/F_2(measured). Equation 9 is used to solve problems in Examples H and I.

EXAMPLE H: BENCH VISE AS A SCREW

Given: The screw in a bench vise has a pitch of 0.25 inch. A worker applies a tangential force of 30 lb on the handle at a point 8 inches from the axis of the screw.

Find: (a) The M_i of the vise, (b) the force that could be exerted by the vise jaws (under ideal conditions), and (c) the % efficiency of the vise. (When the force exerted by the vise jaws is actually measured, it is found to be only 4000 lb.)

Solution:

(a) $$M_i = \frac{2\pi r_2}{x_1}$$

$$= 2(3.14)(8\text{ in})/0.25\text{ in}$$

$$M_i = 201$$

(b) $$F_1 = M_i F_2$$

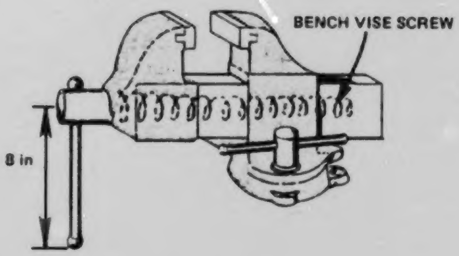
$$= (201)(30\text{ lb})$$

$$F_1 = 6030\text{ lb}$$

(c) $$\eta = \frac{F_1(\text{actual})}{F_1(\text{ideal})}(100)$$

$$= \left(\frac{4000\text{ lb}}{6030\text{ lb}}\right)(100)$$

$$\eta = 66.3\%$$



In the ordinary use of a screwdriver, the radius r_2 is the radius of the driver handle, as shown in Figure 11.

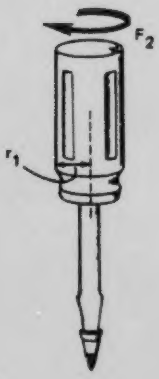


FIG. 11

EXAMPLE I: SCREW JACKS USED IN LIFTING A BUILDING

Given: Four screw jacks are placed under the corners of a square structure weighing 25 tons. The screw jacks are identical, each with a pitch of 0.20 in, and a handle 2.5 ft long, measured from the screw axis.

Find: (a) The M_i of each screw jack, (b) the force that would have to be applied to each screw jack to lift the load when all four are working together, assuming 100% efficiency, and (c) the force if the screw jacks are only 30% efficient.

Solution: (a)

$$M_i = \frac{2\pi r_2}{x_1}$$

$$= 2(3.14)(30\ \text{in})/0.20\ \text{in}$$

$$M_i = 942$$

(b) Each screw jack must take 1/4 the total load, or 25/4 tons = 6.25 tons. This value is (6.25)(2000) = 12,500 lb each. Then —

$$F_2 = \frac{F_1}{M_i}$$

$$= 12{,}500/942$$

$$F_2 = 13.3\ \text{lb}$$

(c)

$$\eta = \frac{F_2(\text{ideal})}{F_2(\text{actual})}$$

$$= \frac{F_2(\text{ideal})}{\eta}$$

$$= 13.3\ \text{lb}/0.30$$

$$\eta = 44.3\ \text{lb}$$

THE WHEEL AND THE AXLE

When a wheel and axle are connected rigidly in such a manner that they turn together, another type of force transformer results in the rotational mechanical system. For example, Figure 12 shows two wheels of different radii, around which ropes are wound. One rope holds the load; the other is pulled by the input force F_2. To determine the M_i of this simple device, consider one revolution of the axle. The rope applying the input force moves a distance equal to the circumference of the large wheel, or $2\pi r_2$. The rope holding the weight or load moves through a distance equal to the circumference of the smaller wheel, or $2\pi r_1$.

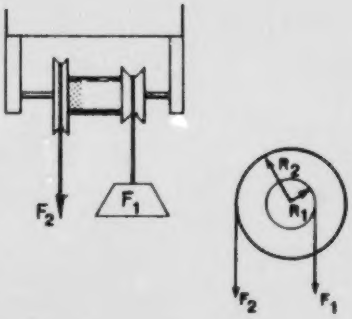


FIG. 12

When the principles of Equation 4 are applied, Equation 10 is derived to resolve this wheel-and-axle problem. An application is shown in Example J.

$$M_i = \frac{x_2}{x_1} = \frac{2\pi r_2}{2\pi r_1} = \frac{r_2}{r_1} \tag{10}$$

where

r_1 = radius of load wheel.
r_2 = radius of source, or drive, wheel.

EXAMPLE J: MECHANICAL ADVANTAGE OF A WHEEL AND AXLE

Given: A bucket holding 30 lb of water is raised from a well by the type of wheel and axle shown. The turning handle has a length of 1.5 ft, and the rope holding the bucket is wrapped around a shaft of 6 inch diameter.

Find: The M_i and the required input force at the handle.

Solution:

$$M_i = \frac{r_2}{r_1}$$

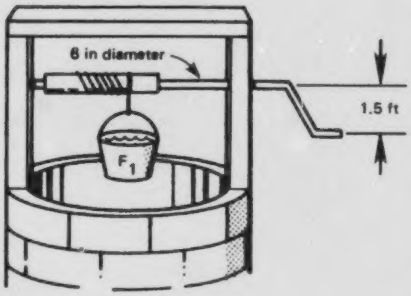
$$= \frac{18}{3}$$

$$M_i = 6$$

$$F_2 = \frac{F_1}{M_i}$$

$$= \frac{30}{6}$$

$$F_2 = 5 \text{ lb}$$



FLUIDAL FORCE TRANSFORMERS - THE HYDRAULIC LIFT

An example of a force transformer in the fluid system is the useful hydraulic lift. A diagram of a hydraulic jack is given in Figure 13, which illustrates a source of energy coupled to a load through an incompressible fluid-like oil. A heavy load, such as a 4000-lb automobile, can be lifted at the load piston by a much smaller force applied at the source piston. Again, this feat is possible because the applied force F_2 must move through a larger distance than the load. Suppose the source piston is pushed down a distance x_2. How far does the load piston rise? Since the fluid is practically incompressible, the volume of fluid V_2 displaced on the source side must show up on the load side V_1. This relationship can be written —

$$V_2 = V_1$$

or

$$x_2A_2 = x_1A_1$$

where

V_1 = volume of fluid displaced on the load side.
V_2 = volume of fluid displaced on the source side.
A_1 = cross-sectional area of piston on the load side.
A_2 = cross-sectional area of piston on the source side.

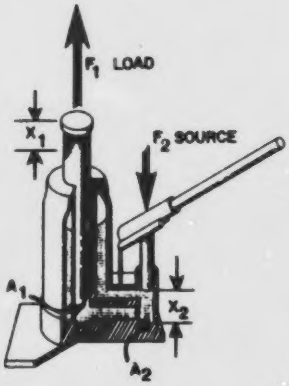


FIG. 13

The last equation leads to this proportion:

$$\frac{x_2}{x_1} = \frac{A_1}{A_2}$$

The mechanical advantage can be derived as an expression of the work in a fluid system as the product of the pressure and volume displacement; however, use of Equation 4 is simpler and more straightforward:

$$M_I = \frac{x_2}{x_1}$$

As shown in the previous equation, the ratio of displacement x_2/x_1 is the same as the ratio of areas, A_1/A_2. The M_I then can be written as Equation 11 for a hydraulic lift.

$$M_I = \frac{A_1}{A_2} \qquad (11)$$

Since little friction is involved in the operation of the hydraulic lift, the actual M_i can be expected to approach the ideal value given by this equation. An application is shown in Example K.

EXAMPLE K: MECHANICAL ADVANTAGE OF A HYDRAULIC LIFT

Given: A hydraulic lift contains a load piston 9 inches in diameter and a pump piston 1 inch in diameter.

Find: (a) The M_i of the lift under ideal conditions and (b) the force on the pump piston required to lift a total load of 5000 lb.

Solution:

(a)

$$M_i = \frac{A_1}{A_2} = \frac{\pi r_1{}^2}{\pi r_2{}^2} = \frac{r_1{}^2}{r_2{}^2}$$

$$= (4.5\ \text{in})^2/(0.5\ \text{in})^2$$

$$M_i = 81$$

(b)

$$F_2 = \frac{F_1}{M_i}$$

$$= 5000\ \text{lb}/81$$

$$F_2 = 61.7\ \text{lb}$$

ELECTRICAL TRANSFORMERS

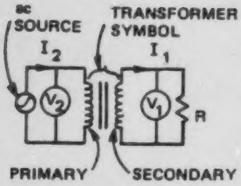


FIG. 14

In the electrical system, the forcelike quantity is voltage. The electrical transformer is a device that can step up or can step down voltage at the expense of current. The source in this case is producing alternating current, and the load is some device that consumes electrical energy. In Figure 14, the load is shown as a simple resistor R. (Notice the symbol for the transformer used in drawing electrical circuits.) V_2 and V_1 represent input and output voltages, respectively. Voltmeters placed in the positions shown would record the appropriate voltages. The transformer itself usually is a rectangular iron core with two windings — the primary windings on the source side and the secondary windings on the load side. Figure 15 consists of a simple drawing of a transformer in the same circuit.

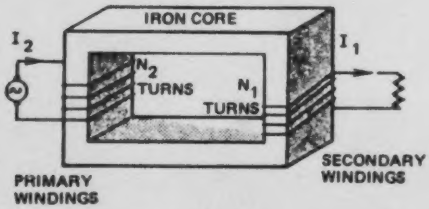


FIG. 15

As with the belt drive, source energy is fed to the electrical transformer at a continuous rate. Thus, in an ideal transformer, in which no energy losses occur, the following equality is true for power input = power output:

$$V_2I_2 = V_1I_1$$

where

V_1 = voltage output.
V_2 = voltage input.
I_1 = current output.
I_2 = current input.

(Refer to Concept Module 6-0 for the expression of power in the electrical system.)

The complete theory of the transformer is complicated, since it involves the magnetic fields produced within the iron core by the changing current in the windings. However, the theory shows that the voltage across the primary and secondary windings is proportional to the number of turns, or windings, on each:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

or

$$V_1 = \left(\frac{N_1}{N_2}\right) V_2$$

where

N_1 = secondary winding turns.
N_2 = primary winding turns.

The voltage, or forcelike quantity, at the source clearly is multiplied by the ratio of turns N_1/N_2 to give the voltage at the load. This ratio then must be the M_i of the transformer, as summarized in Equation 12.

$$M_i = \frac{N_1}{N_2} \qquad (12)$$

Use of Equation 12 is exemplified in Example L.

The electrical transformer can step up (increase) or step down (decrease) the primary, or source, voltage, depending upon the ratio of secondary-to-primary turns of the windings. This formula is correct for ideal transformers. Actual transformers waste some energy through resistance heating in the windings, production of eddy currents, and hysteresis losses. All these losses are analogous to friction in the mechanical system.

Because in the ideal case, power input = power output, or $V_2I_2 = V_1I_1$, a step-up of voltage leads to a step-down of current, and vice versa. Thus, an electrical transformer also can serve as a current transformer. It can be used, for example, to produce a large current from a smaller one at the expense of voltage. (See Examples L and M.)

EXAMPLE L: VOLTAGE STEP-DOWN IN A TRANSFORMER

Given: An electrical transformer has 200 turns on the primary and 20 turns on the secondary, and a source voltage is 120 volts.

Find: Voltage available at the load.

Solution:

$$V_1 = (M_i)\ V_2$$

$$= \left(\frac{N_1}{N_2}\right) V_2$$

$$= (20/200)\ 120 \text{ volts}$$

$$V_1 = 12 \text{ volts}$$

EXAMPLE M: STEP-UP OF CURRENT IN A TRANSFORMER

Given: The input current to a transformer is 1 amp at 2000 volts.

Find: The output current at 100 volts, assuming the efficiency is 100%.

Solution: In an ideal transformer, power input = power output. Therefore —

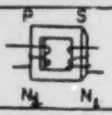
$$I_2 V_2 = I_1 V_1$$

$$(1 \text{ amp})(2000 \text{ volts}) = (I_1)(100 \text{ volts})$$

$$I_1 = 20 \text{ amps}$$

Table 1 summarizes the cases for voltage and current multiplication.

TABLE 1. VOLTAGE AND CURRENT RELATIONSHIPS IN A TRANSFORMER

Number of Turns		Voltage	Current	M_i (Voltage Multiplication)
	$N_1 < N_2$	$V_1 < V_2$	$I_1 > I_2$	$N_1/N_2 < 1$
	$N_1 > N_2$	$V_1 > V_2$	$I_1 < I_2$	$N_1/N_2 > 1$

SUMMARY

Now that a number of force transformers, or simple machines, have been considered in detail, to summarize their major characteristics and emphasize their basic unity may be helpful.

The force transformer is a coupling device between an energy source and an energy load. The transformer changes, or transforms, input values of forcelike, displacementlike, and ratelike quantities into different values at the output. There is, however, a fundamental law governing these changes. The law states that where no energy is lost through frictional effects (ideal transformer), the energy input equals the energy output. Mathematically speaking, the product of forcelike and displacementlike quantities has the same value at input and output. Furthermore, where the transformer operates continuously so that one can speak of rate of work, or power, one can state that in the ideal case, the power input equals the power output. Equivalently, the product of forcelike and ratelike quantities has the same value at input and at output.

In the actual case, some of the energy or power input is used up by frictional affect, so that the energy and power output is less than the corresponding input, as is summarized in Figure 16.

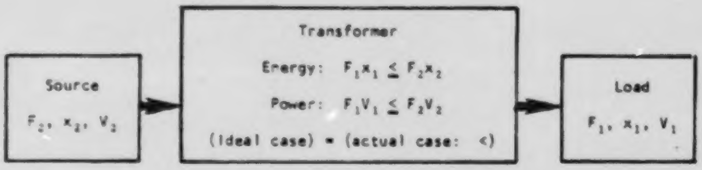

Source
F_2, x_2, V_2
Transformer
Energy: $F_1x_1 \leq F_2x_2$
Power: $F_1V_1 \leq F_2V_2$
(ideal case) = (actual case: <)
Load
F_1, x_1, V_1

FIG. 16

A characteristic of the force transformer is that where a force advantage is achieved ($F_1 > F_2$), it is achieved at the expense of a distance or velocity disadvantage ($x_1 < x_2$ or $V_1 < V_2$). Only in this way can energy and power products be the same at input and output. The ratio F_1/F_2 — or alternately, the ratio x_2/x_1 — is called the "mechanical advantage" of the transformer. Clearly, the "advantage" is one of force, as long as the M_i is greater than 1.

The foregoing statements apply to all energy systems, as indicated in Table 2, which includes all fundamental energy systems except the thermal system. A few minutes spent studying this table should reinforce the analogous nature of the force transformer in these different energy systems. The table also lists, for the sake of convenience, the M_i for each transformer.

TABLE 2. TRANSFORMERS FOR DIFFERENT ENERGY SYSTEMS

System	Typical Transformers	M_i (ideal)	Forcelike, Displacementlike, Ratelike Quantities	Energy and Power Relationships (= for ideal) (< for actual)
Mechanical translational	Pulley	No. of supporting strand	Source: F_2, x_2, V_2	Energy: $F_1x_1 \leq F_2x_2$
	Lever	ℓ_2/ℓ_1 (lever arms)	Load: F_1, x_1, V_1	Power: $F_1V_1 \leq F_2V_2$
	Inclined plane	Length/rise		
Mechanical rotational	Screw	$2\pi r_2$/pitch		
	Wheel and axle	r_2/r_1 (radii)	Source: $\tau_2, \theta_2, \omega_2$ Load: $\tau_2, \theta_1, \omega_1$	Energy: $\tau_1\Delta\theta_1 \leq \tau_2\Delta\theta_2$ Power: $\tau_1\omega_1 \leq \tau_2\omega_2$
Fluid	Hydraulic jack	A_1/A_2 (areas)	Source: p_2, V_2, V_2 Load: p_1, V_1, V_1	Energy: $p_1\Delta V_1 \leq p_2\Delta V_2$ Power: $p_1V_1 \leq p_2V_2$
Electrical	Voltage Transformer	N_1/N_2 (turns)	Source: V_2, Q_2, I_2 Load: V_2, Q_1, I_1	Energy: $V_1Q_1 \leq V_2Q_2$ Power: $V_1I_1 \leq V_2I_2$

MECHANICAL ADVANTAGE OF A LEVER

MATERIALS

Set of weights up to about 250 grams.
Meter stick prepared with holes drilled to accommodate a large nail, every 10 cm.
Ring stand and clamps.
Assortment of spring balances.

PROCEDURE

Three basic types of levers will be tested in this demonstration. In each case, the lever will be a meter stick having holes drilled every 10 centimeters along its length. Holes will be large enough to accommodate a nail that can act as a pivot. The sharp end of the nail can be clamped onto a ringstand, as shown in Figure 17. In each of the three cases outlined on the following page, hang an arbitrary load with spring or weight holder at an arbitrary point, and then determine experimentally the source force required to produce balance. The required balancing force is applied either by hanging weights or by a spring balance. Do not neglect the weight holder, if one is used. The diagrams below will make each of the three cases clear. Notice that the cases differ in the relative positions of load (L), source (S), and pivot or fulcrum (F). Complete Data Table I, which includes the data and values calculated from the data.

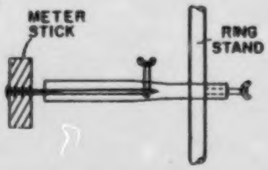
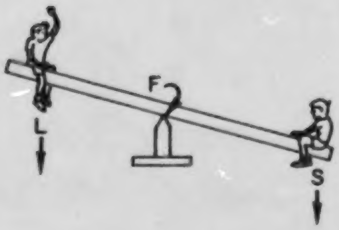
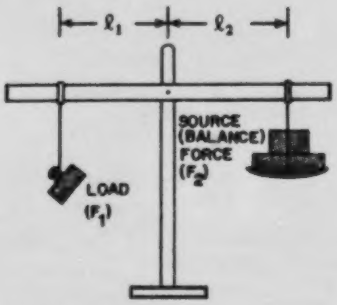


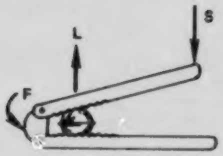
FIG. 17

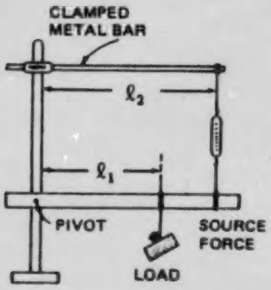
Case 1: Wooden-beam Lever



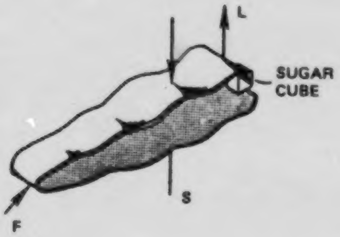


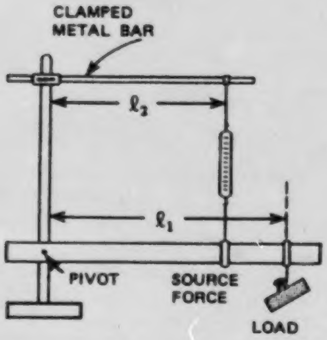
Case 2: Nut-cracker Lever




CLAMPED METAL BAR
ℓ_2
ℓ_1
PIVOT
SOURCE FORCE
LOAD

Case 3: Chemical-tongs Lever


L
SUGAR CUBE
S
F


CLAMPED METAL BAR
ℓ_2
ℓ_1
PIVOT
SOURCE FORCE
LOAD

DATA TABLE 1.

	Source Lever Arm ℓ_2 (cm)	Load Lever Arm ℓ_1 (cm)	Load Force F_1 (dynes)	Source Force F_2 (dynes)	M_i ideal ℓ_2/ℓ_1	M_a actual F_1/F_2	$\eta = \frac{M_a}{M_i}(100)$
Case 1							
Case 2							
Case 3							

MECHANICAL ADVANTAGE OF AN INCLINED PLANE

MATERIALS

Smooth board about 1 meter long.
Small car that can carry various weights.
Fixed pulley that can be clamped to the end of a board.

PROCEDURE

Determine the weight (source force) needed just to roll the car (load) up the incline, as shown in Figure 18. Make several trials with different weights in the car. Add weights gradually to weight holder until car just rolls up the incline without acceleration. Add the weight of the car to the weight of the load it carries. Measure the incline length and rise, and complete Data Table 2.

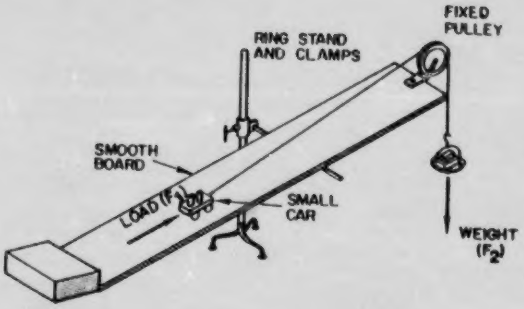


FIG. 18

DATA TABLE 2.

Length of Incline (cm)	Rise of Incline (cm)	Load (dynes)	Source Force (dynes)	M_i $\frac{\text{length}}{\text{rise}}$	M_a F_1/F_2	Efficiency

ELECTRIC TRANSFORMER EFFICIENCY

MATERIALS

12-V transformer.
a.c. Milliammeter 0-100 MA.
a.c. Ammeter 0-1 A.
Volt-ohm-milliammeter.
Knife switch.
Clip leads
a.c. power cord.
20 Ω, 10w resistor.

PROCEDURE

1. Construct the circuit shown in Figure 19.

2. Plug in a.c. power cord. Be careful to avoid any bare wires or contacts while the cord is plugged into the outlet.

3. Connect VOM to measure a.c. voltage between points A and B of Figure 19.

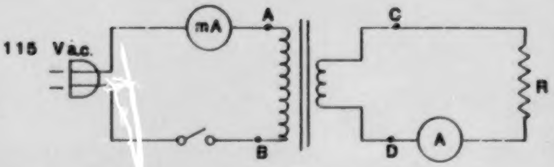


FIG. 19

4. Close switch. Measure and record in Data Table 3 the values for transformer input voltage and current. Open switch.

5. Connect VOM to measure a.c. voltage between points C and D of Figure 19.

6. Close switch. Measure and record values for transformer output voltage and current. Open switch and unplug power cord.

7. Complete Data Table 3.

DATA TABLE 3.

	Input	Output
Voltage		
Current		
Power		

Transformer efficiency = ___________

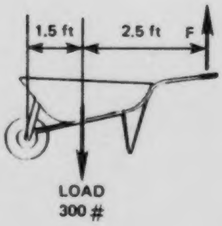
SELF CHECK

1. Describe specific force transformers in the mechanical translational, mechanical rotational, fluidal, and electrical systems; and discuss their fundamental similarity as transformers of forcelike quantities.

2. Define:

   Ideal mechanical advantage.
   Actual mechanical advantage.
   Efficiency.

3. A wheelbarrow contains 300 lb of sand. If the distance from the center of the load to the wheel is 1.5 ft, and the distance from the handle to the wheel is 4 ft, what lifting force F must be applied at the handle to move the sand?



4. The total weight of skiers and equipment on a ski tow is 3000 lb. The cable of the ski tow pulls this load at an incline of 25°. What is the tension in the cable if frictional forces are negligible?

5. A wheel and axle is used to lift a 2400-lb load. The wheel diameter is 60 inches, and the axle diameter is 5 inches. The operator must exert a force of 250 lb to lift the load. Determine:

   Ideal mechanical advantage (M_i).
   Actual mechanical advantage (M_a).
   Efficiency of the transformer.

6. A screw jack is being used to lift a load of 2 tons. The screw pitch is 0.25 inches, and handle length 20 inches. In the ideal case, what effort is needed at the end of the handle? If the efficiency is 20%, what effort is needed?

7. The two pistons of a hydraulic press have diameters of 4 inches and 64 inches. What force must be applied by the smaller piston to give a compressive force of 50 tons at the larger piston? How far does the small piston travel in moving the large piston 1 inch?

8, A doorbell transformer has primary windings of 720 turns and secondary windings of 48 turns. The source voltage is a 120-volt house circuit. What voltage is delivered to operate the bell?

9. Discuss how the role of resistance in a transformer dissipates energy input and reduces efficiency.

10. Describe the power input and power output of a transformer that operates continuously.

11. What is the lever arm of the applied force for a level with an ideal mechanical advantage of five and a load lever arm of 1.5 feet?

12. What force must be applied to a pulley with an efficiency of 83% if the load force is 200 N and the ideal mechanical advantage is 3?

13. What is the ideal mechanical advantage of a plane inclined at an angle of 30°?

14. What is the angle of inclination of a plane if the ratio of input work to output work is 0.8 and the ratio of input force to output force is 0.4? Is there friction in this system? How can one tell?

15. How much fluid is displaced on the source side of a hydraulic lift if the load piston has a cross-sectional area of 0.3 m^2 and is raised 5 cm?

16. What is the cross-sectional area of the source piston of the lift in problem 15 if the lift has an ideal mechanical advantage of 12?

17. What is the required input force to the source wheel of a wheel and axle if the circumference of the load wheel is 0.5 m and the radius of the drive wheel is 0.4 m, when the load is 20 N?

18. What is the current input of an electrical transformer if the input voltage is 18 V and the power output is 200 watts?

19. What is the input current to an electrical transformer if the ideal mechanical advantage is 6 and the output current is 0.2 Amps?

20. What is the efficiency of a pulley that has an ideal mechanical advantage of 2 and an input force to output force ratio of 0.7?

21. The ideal mechanical advantage of a pulley is used to predict the need for a 25 N input force. What input force is actually needed if the pulley has an efficiency of 85%?

22. What is the maximum load that can be pushed up an inclined plane which has an ideal mechanical advantage of 4 if the applied force is 55 N?

23. A lever can be used to lift a 200 kg mass when the input force is 350 N. If the ideal mechanical advantage of the lever system is 6, what is the efficiency of the system?

24. The ideal mechanical advantage of a hydraulic lift is 5. What is the ratio of the radius of the piston on the load side to that of the source piston?

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8

ENERGY CONVERTORS

CHAPTER 9-0

Our modern technological society depends upon the production, distribution, and consumption of large amounts of energy. These processes are made possible by a group of devices that can be classified as "energy convertors." These devices accept input energy in one energy system and deliver output energy in another system. Some of the input energy is lost, because energy convertors are not one-hundred percent efficient. This module discusses (1) a variety of energy convertors, important in the conversion and application of energy and (2) the efficiency of individual energy convertors and systems.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Describe energy convertors in general terms that apply to all energy-conversion devices.
- Describe the operation of the following energy convertors:

  Vane pump.
  Electric generator.
  Turbine.
  Electric motor.
  Electric heater.
  Internal combustion engine.
  Boiler.
  Solar collector.
- Given two of the following quantities, determine the third:

  Input energy.
  Output energy.
  Efficiency.
- Given the efficiency of all energy convertors used in an energy-conversion system, determine overall system efficiency.

DISCUSSION

An energy convertor is a device that accepts energy input in one form and delivers energy in a different form at the output. For example, an electric fan actually is composed of two energy-converting devices. The electric motor converts electrical energy to rotational mechanical energy, and the fan blades convert this mechanical energy to the fluidal energy of moving air.

According to the "law of conservation of energy," energy neither can be created, nor destroyed. Thus, energy convertors actually do not create energy; they only change the input energy to another form. Another consequence of energy conservation is that all input energy must appear as output energy in one form or another. Some of this output energy always appears as heat energy because of resistance in the system. If heat energy is not the desired form of the output, it represents loss in the system. The efficiency of an energy convertor is given by Equation 1.

$$\eta = \frac{E_{out}}{E_{in}} (100) \qquad (1)$$

where

η = efficiency.
E_{in} = input energy.
E_{out} = useful output energy.

Energy convertors also can be considered as power convertors, since some time always is involved in the energy-conversion process. In some cases, to discuss input and output powers is convenient, especially when energy convertors are in steady-state operation. Equation 2 states the relationship of energy, power, and time.

$$P = \frac{E}{t} \qquad (2)$$

where

P = power.
E = energy.
t = time.

A wide variety of energy-conversion devices are in common use today, and most energy used in the United States is changed in form several times before its final application. The electrical energy used to operate the fan mentioned earlier originally was chemical energy contained in a fuel. The fuel was burned to produce thermal energy, which was converted to fluidal energy in high-pressure steam. A turbine converted the steam to mechanical energy. Our entire society depends upon our ability to convert energy from one form to another in a controlled and efficient manner. Figure 1 is an energy-flow chart for the United States in 1975. Notice that 55% of the energy consumed in this country in 1975 was wasted. With increasing energy demands and decreasing supplies, the improvement of efficiency in energy convertors is of prime importance.

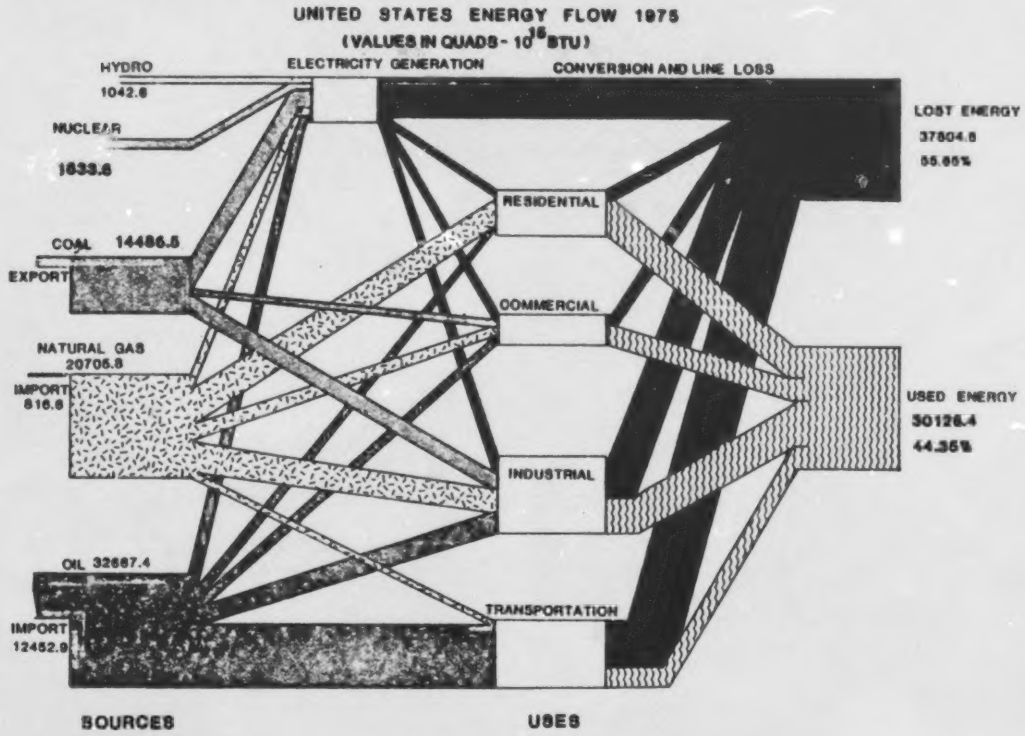


FIG. 1

Figure 2 is a chart showing the energy consumption in the United States during this century and the project supply and demand during the next century. This Figure shows the importance of improving energy convertors and of developing new types of energy convertors that can make efficient use of "ultimate energy sources.

Table 1 is a listing of the energy convertors discussed in the remainder of this module. Some of them have been in common use for many years, while others currently are in research and development. In this module, optical (radiant) energy is added as an additional and important form of energy. Energy convertors are discussed in the order of the form of their input energy.

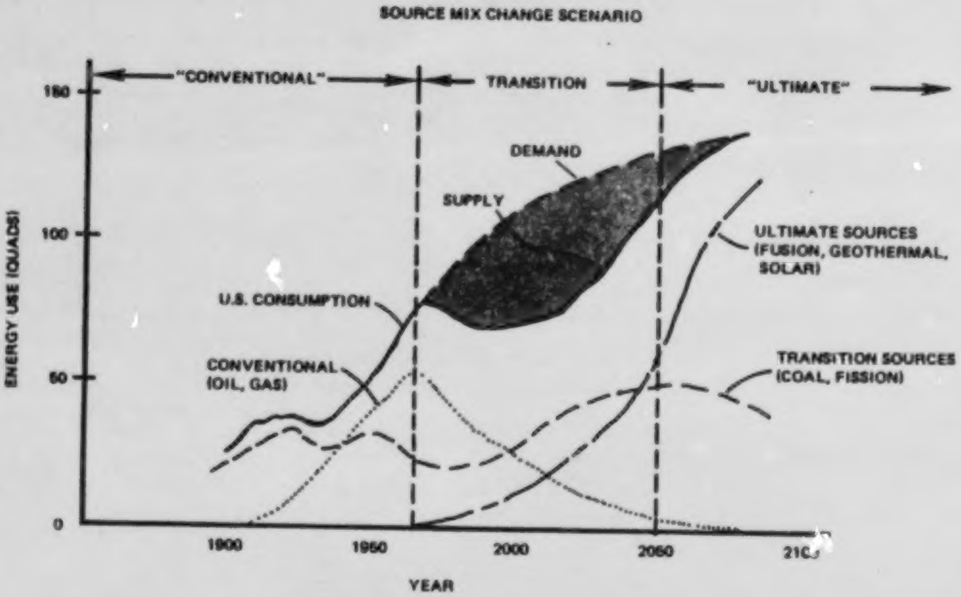


FIG. 2

TABLE 1. IMPORTANT ENERGY CONVERTORS

	Energy Convertors	OUTPUT ENERGY				
		Mechanical	Fluidal	Electrical	Thermal	Optical
INPUT ENERGY	Mechanical	▨	Pump Fan	Generator	▨	▨
	Fluidal	Turbine Windmill	▨	Magneto-hydrodynamic Generator	▨	▨
	Electrical	Motor	▨	▨	Heating Devices	Lighting Devices
	Thermal	Combustion Engines	Steam Generator	Thermo-electric Generators	▨	▨
	Optical	▨	▨	Solar Cells	Solar Collectors	▨

CONVERTORS OF MECHANICAL INPUT ENERGY

Mechanical energy is the energy of mass in motion and, as such, is one of the most important final forms of useful energy. However, mechanical energy also is an important intermediate step in several energy-conversion processes. Devices used to convert mechanical energy to other useful forms of energy are discussed below.

FLUIDAL OUTPUT ENERGY

Figure 3 shows a vane pump used to convert mechanical rotational power to fluidal power. This type of pump can be used with either liquids or gases. The fluid is pulled into the inlet by the partial vacuum created as the vanes move upward from the inlet. The vanes are spring-loaded to maintain a seal against the pump housing. The fluid then is trapped between two vanes and forced through the pump to the outlet. This is only one possible pump configuration; there are several others. Figure 4 shows the characteristic curves of an axial piston pump. Notice that such pumps are highly efficient devices.

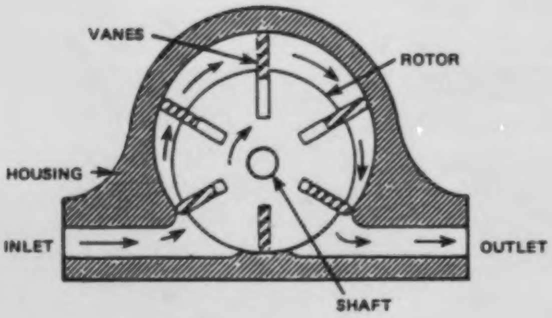


FIG. 3

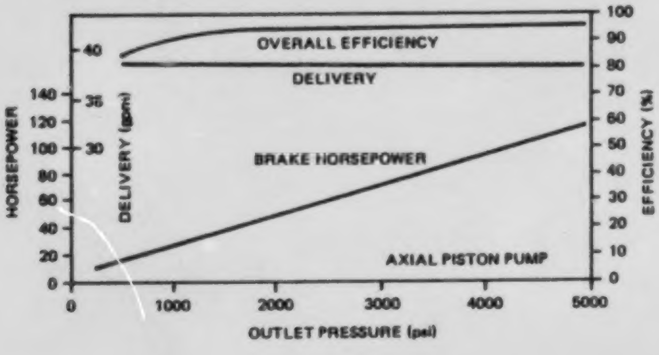


FIG. 4

Fans are devices used to convert mechanical input energy to the fluidal energy of moving air. Figure 5 shows a propeller fan and the air flow pattern produced. A centrifugal fan and its air flow are shown in Figure 6.

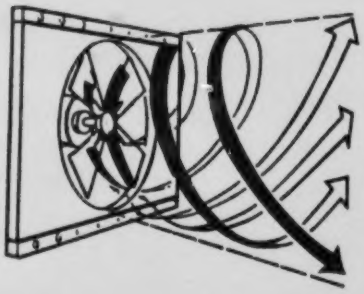


FIG. 5

ELECTRICAL OUTPUT ENERGY

When an electrical conductor is passed through a magnetic field perpendicular to the field lines, a force is present on electrons in the conductor. This force causes them to flow, as shown in Figure 7a. Figure 7b shows this principle applied to an electrical generator. The loop of wire is rotated in a magnetic field by input mechanical energy. This energy is converted to electrical energy in the form of voltage and current in the loop. The electrical energy can be transmitted over long distances, to be converted to other forms of energy for application.

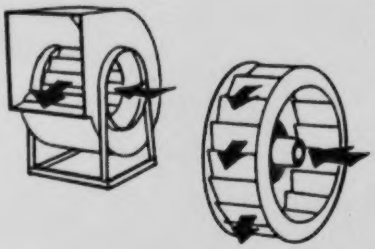


FIG. 6

An alternator is a device in which the coils of wire are stationary, and a magnet is rotated inside them. Electrical generating plants produce electrical power using alternators in which the magnetic field is produced by a current flowing through rotating coils. These large alternators have an efficiency of 60-70 percent. The energy loss is caused by mechanical friction, air resistance, and electrical losses.

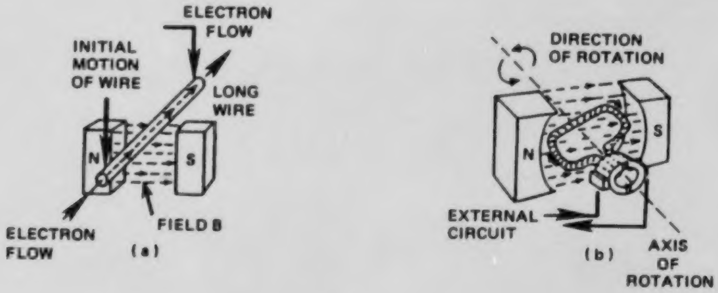


FIG. 7

CONVERTORS OF FLUIDAL INPUT ENERGY

MECHANICAL OUTPUT ENERGY

Conversion of fluidal energy to mechanical energy is essential to the large-scale generation of electrical energy. All commercial electrical energy produced in the United States is fluidal energy at some point in its production. In most cases, this energy is contained in high-pressure steam. Figure 8 shows a section of a turbine used to convert this fluidal energy to rotational mechanical energy. Steam with high velocity and pressure strikes the movable blades of the turbine, exerting a force that causes the turbine to rotate. Several stages (additional sets of turbine blades) generally are used to extract more energy from the steam. The steam turbines used in modern electrical generation stations typically operate with efficiencies of 80-85 percent.

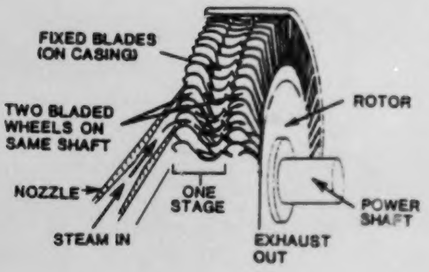


FIG. 8

Windmills also are devices for converting fluidal energy to mechanical energy, but they work with much lower fluid velocities than do most turbines. Many new designs for windmills have been developed and tested in recent years. The theoretical maximum efficiency of a windmill in changing the kinetic energy of moving air to rotational mechanical energy is 59 percent. The latest high-speed models are capable of actual efficiences of about 50 percent. Example A shows the calculation of windmill efficiency using Equation 1.

EXAMPLE A: CALCULATING EFFICIENCY

Given: A windmill has a 25-foot diameter aluminum rotor; there is a wind velocity of 28 mi/h; and 20 kilowatts of electrical power is generated.

Find: Efficiency (from mechanical energy input of wind to electrical-energy output).

Solution: Convert to metric units. 25-foot diameter (d) becomes 7.62 m; 28 mi/h velocity becomes 12.62 m/s; mass of air that moves through the circular area covered by the blades in one second must be calculated:

Blade area x air density x air velocity, or —

$$m = \frac{\pi}{4}d^2\rho v$$

where

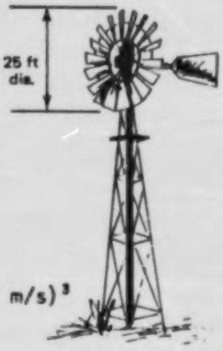
m = mass of air that moves through area of blades.

d = diameter of rotor.

ρ = density of air (1.29 kg/m³).

v = wind velocity in m/s.

The kinetic energy of this air is 1/2 mv^2. Thus, total energy brought by the wind to the blades in one second can be calculated as follows:



$$E_T = \left(\frac{1}{2}\right) \frac{\pi}{4}d^2\rho v \ (v^2)$$

$$= \frac{1}{2}\left(\frac{3.14}{4}\right)(7.62\text{ m})^2(1.29\text{ kg/m}^3)(12.52\text{ m/s})^3$$

$$E_T = 5.77 \times 10^4 \text{ joules (in one second)}$$

(Note: There are 30.48 cm in 1 foot, and 44.7 cm/s in mi/h.)

Electrical energy generated in one second is 20,000 joules (20 kW).

$$\text{Efficiency:}\quad \eta = \frac{E_{out}}{E_{in}}(100)$$

$$= \frac{20,000}{57,900}$$

$$= 0.345$$

$$\eta = 34.5\%$$

(Remember: 1 joule = 10^7 ergs.)

ELECTRICAL OUTPUT ENERGY

A new kind of kinetic energy convertor, which may have practical importance for generation of electricity in the 1980's, is the magnetohydrodynamic generator. The design of this energy convertor is based upon the flow of hot, electrically-conductive gases through a magnetic field. These gases used are "plasmas"; that is, they consist of free electrons and positive ions.

There is a physical law that states, "The passage of such a gas through a magnetic field produces a voltage perpendicular to both the gas flow and the magnetic field." (Refer to Figure 9.)

The gas flows in a duct through a strong magnetic field, the field producing a voltage between two electrodes. Generation of electrical energy is accomplished by a cooling of the gas as it expands in the duct. After cooling, the stream of gas is compressed, reheated, and returned to the generator.

A magnetohydrodynamic generator can operate with higher efficiencies than can a conventional electrical power plant. It has no moving parts; thus, its operation is more reliable than that of a conventional generator. Severe problems yet must be overcome, however. If they can be overcome, magnetohydrodynamic generators offer an important source of electrical energy.

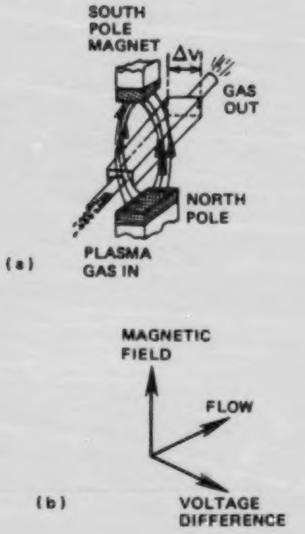


FIG. 9

CONVERTORS OF ELECTRICAL INPUT ENERGY

MECHANICAL OUTPUT ENERGY

In an electrical generator, a current is produced by the movement of a conductor through a magnetic field or by the movement of a magnetic field past a fixed conductor. In an electric motor, the forces that produce motion in conductors result from current flowing in the conductors perpendicular to a magnetic field, as illustrated in Figure 10. The basic construction of motors and generators is the same; in fact, many motors will operate well as generators if mechanical input energy is supplied.

Many types of electric motors have been developed for efficient operation in a variety of applications. Each motor operates at its maximum efficiency at the power for which it is designed. Figure 11 shows efficiency as a function of power delivered for a typical industrial motor.

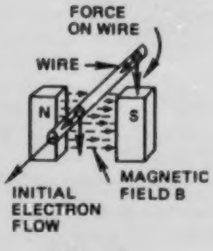


FIG. 10

Three important factors can conserve energy consumed by electric motors:

(1) A motor operates more efficiently at full, or near, capacity; that is, a motor just large enough to handle the load is much more efficient than a large one operating at far less than full capacity.

(2) External capacitors, mounted at the motor or at the distribution equip-

ment, can reduce power consumption of large a.c. motors.

(3) Synchronous motors consume less power than induction motors; therefore, their use can conserve energy. Some of them require both a.c. and d.c.; they operate at up to 1200 rpm and up to 300 hp. The initial cost for synchronous motors often is less than for induction motors.

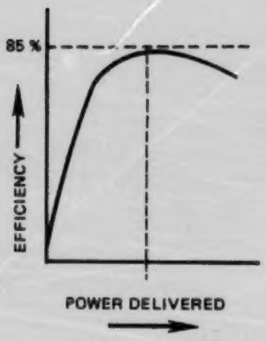


FIG. 11

THERMAL OUTPUT ENERGY

A wide variety of common devices convert electrical energy to thermal energy. These devices include heaters, coffee pots, clothes dryers, industrial furnaces, and welders. In all cases, the thermal energy is produced by current flowing through a resistance. If the electrical load is a pure resistor, the conversion efficiency is 100 percent, and the thermal power can be calculated with Equation 3.

$$P = IV \qquad (3)$$

where

P = power (watts).
I = current (amperes).
V = potential difference (volts).

Example B shows the use of this equation in solving a problem.

EXAMPLE B: THERMAL POWER OF A COFFEEPOT

Given: A coffee pot operating at 110 V requires a current of 18.18 amperes.

Find: The thermal power of the coffee pot.

Solution:

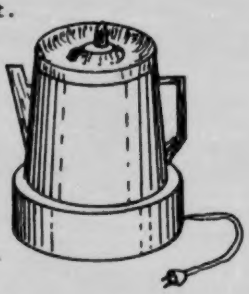
$P = IV$

$= (18.18 \text{ A})\ (110 \text{ V})$

$P = 2000 \text{ W}$

Converting to thermal units:

1 calorie = 4.186 joules



Thus, 1 cal/s = 4.186 W

$$P = (2000\ W)\ (\frac{1\ cal/s}{4.186\ W})$$

$$P = 477\ cal/s$$

OPTICAL OUTPUT ENERGY

An important application of electrical energy is in the production of light. Devices that convert electrical energy to optical energy include television sets, light-emitting diodes, and lasers, as well as both fluorescent and incandescent lights. Incandescent lights produce optical energy when tungsten wires within them are heated to high temperatures. A large portion of the energy they consume, therefore, is radiated as heat energy. Fluorescent lights do not depend upon thermal emission, and are much more efficient.

CONVERTORS OF THERMAL INPUT ENERGY

MECHANICAL OUTPUT ENERGY

Several types of heat engines have been developed for the conversion of thermal energy to mechanical energy. The most widely-used are the internal-combustion engines used in transportation. All these engines make use of an intermediate step in which thermal energy is changed to the fluidal energy of a gas and then to mechanical motion. Internal-combustion engines have an efficiency of about 30 percent.

FLUIDAL OUTPUT ENERGY

Most of the energy consumed in the United States is produced initially in the form of hot combustion gases. Internal-combustion engines combine these gases and convert the resulting fluidal energy directly to mechanical energy. In other cases, such as in electrical power production, thermal energy is converted to fluidal energy in a boiler. Modern boilers consist of a maze of tubes filled with water and steam. These tubes are surrounded by the hot combustion gases, and thermal energy is conducted through their walls to the water inside. Every effort is made to extract as much thermal energy as possible from the hot gases. Figure 12 shows some of the design features of a modern boiler. The intake air for combustion and the input water to the boiler tubes are heated by escaping gas. A superheater at the output of the boiler increases both steam pressure and efficiency. Even with these innovations, boilers are only about 70 percent efficient.

ELECTRICAL OUTPUT ENERGY

Thermoelectricity is the direct conversion of heat energy to electrical energy. When two different electrical conductors are joined, a small voltage is produced across the junction. The magnitude of this voltage depends upon the temperature of the junction. Figure 13 shows the schematic of a thermoelectric generator. The two junctions are at different temperatures (T_1 and T_2); therefore, the potential across the junction is not the same. This difference results in a net voltage across points A and B of the Figure. The voltage produced by thermoelectric generators is small, but large currents can be produced.

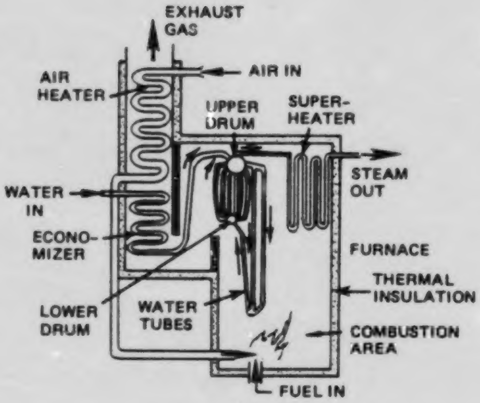

EXHAUST GAS
AIR IN
AIR HEATER
UPPER DRUM
SUPER-HEATER
STEAM OUT
WATER IN
ECONO-MIZER
FURNACE
THERMAL INSULATION
LOWER DRUM
WATER TUBES
COMBUSTION AREA
FUEL IN

FIG. 12

Thermoelectric generators exhibit an efficiency of only about 10 percent and have the added disadvantage of high cost. Their only present application in this country, therefore, is in the production of electrical power for spacecraft.

CONVERTORS OF OPTICAL INPUT ENERGY

ELECTRICAL OUTPUT ENERGY

Photoelectric cells, often called "solar cells," are devices made of semiconductors, such as silicon, that directly convert a portion of the optical energy striking them into electrical energy. Solar cells have long been used to convert the energy of sunlight to electrical energy for use in satellites, but the cost of the devices and their low efficiency (typically less than 10 percent) has made large-scale use uneconomical.

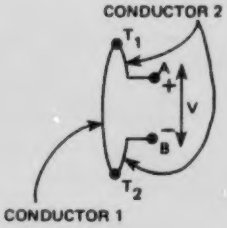

CONDUCTOR 2
T_1
A
+
V
−
B
T_2
CONDUCTOR 1

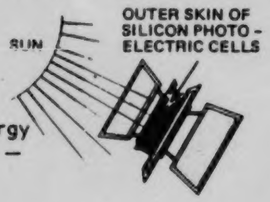
FIG. 13

Example C illustrates the determination of the size of a photoelectric cell array for use in electrical power production for a satellite.

EXAMPLE C: CALCULATION OF ELECTRICAL ENERGY USING SOLAR CELLS

Given: Silicon photoelectric energy convertors are used on a satellite to generate electricity with 8% efficiency. The silicon photocells always are pointed directly at the sun. (Above the earth's atmosphere, the sun delivers radiant power per unit area of 0.14 watts/cm².

Find: Area of solar cells needed to generate 12 watts of electrical power.



Solution: Required watts = area x light energy x efficiency, or —

$$W = AE_L\eta$$

$$12 \text{ watts} = (A \text{ cm}^2)(0.14 \text{ watts/cm}^2)(0.08)$$

$$A = \frac{12}{0.14 \times 0.08} \simeq 1.07 \times 10^3 \text{ cm}^2$$

$$A = 1070 \text{ cm}^2$$

THERMAL OUTPUT ENERGY

Sunlight, streaming onto the earth's surface, represents a substantial source of energy. All one needs is a suitable "bucket" in which to catch it. Such buckets are under development. A solar house, with large windows facing south, collects solar energy and uses it directly as heat. Designs for solar water heaters have been under intensive development. Central solar-powered generating stations (large banks of movable mirrors track the sun and focus energy on a tower containing water) are being investigated. Solar-heated water is converted to steam used to drive a turbine to generate electricity.

SUMMARY

Energy convertors are devices that convert energy from one energy system to another. In all cases except for the direct conversion of electrical energy to thermal energy, some of the input energy appears as waste heat. Thus, energy convertors are not 100 percent efficient. The accumulative losses in energy convertors accounts for over half of all the energy consumed in the United States. Example D illustrates a major contribution to energy loss by showing the overall efficiency of a typical electrical generating station.

EXAMPLE D: EFFICIENCY OF AN ELECTRICAL POWER PLANT

Given: The following are the efficiencies of the energy convertors in a typical electrical power plant:

Boiler (chemical energy to thermal energy to fluidal energy) — 70%
Turbine (fluidal energy to mechanical energy) — 80%
Generator (mechanical energy to electrical energy) -- 60%

Find: Overall efficiency.

Solution: The overall efficiency of a series of energy convertors is the product of the individual efficiencies expressed as decimal fractions:

$$\eta_T = \eta_1 \eta_2 \eta_3 \ldots$$

Thus —

$$\eta_T = (0.70)(0.80)(0.60)$$

$$\eta_T = 0.336 = 33.6\%$$

This example illustrates the importance of improving the efficiency of energy convertors.

EFFICIENCY OF AN ELECTRIC WATER PUMP

MATERIALS

a.c. electric water pump.
Wattmeter.
Two large containers of known volume.
Connecting hose.
Stopwatch.

PROCEDURE

This experiment requires energy calculations at both input and output of a combined electrical fluid system.

1. Arrange an electric pump to lift water from a reservoir on floor to a higher level as illustrated in Figure 14.
2. Connect wattmeter in pump circuit, as in Figure 15.
3. With a stopwatch, measure time necessary for pump to fill a relatively large container of known volume. Measure the electrical input power P and liquid head h. Record data in Data Table.
4. Complete Data Table, calculating energy input and useful energy output of system.
5. Calculate overall efficiency of system.

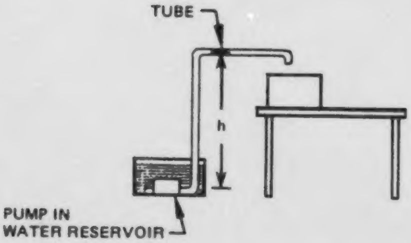


FIG. 14

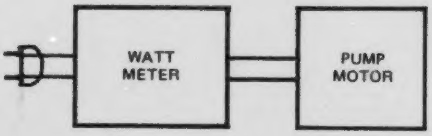


FIG. 15

<table>
<tr><th colspan="2">DATA TABLE</th></tr>
<tr><th>Power Input
(Electrical System)</th><th>Power Output
(Fluid System)</th></tr>
<tr><td>Power input to pump P = ________
Time required to pump water
t = ____________
Input energy Pt = ___________</td><td>Weight density of water
ρ = ___________
Pressure head h = ________
Pressure $p = \rho h$ = ________
Volume of water pumped
ΔV = ___________
Output energy for transferring water $E = p\Delta V$ = ___</td></tr>
<tr><td colspan="2">Efficiency $\% = \frac{\text{energy output}}{\text{energy input}} (100)$ = ______________</td></tr>
</table>

EFFICIENCY OF A PHOTOCELL

MATERIALS

Photocell (United Detector Technology model PIN-10DP or equivalent).
Microscope with calibrated reticle (not needed if area of photoconductor is known).
Milliammeter.
Resistor (50 Ω).
Wiring and clips for electrical connections.

PROCEDURE

In this experiment, the efficiency of conversion of sunlight into electrical energy is measured. A photocell — a convertor of optical energy to electrical energy — is required. (Note: This experiment must be performed on a bright, clear day.)

1. Use microscope with calibrated reticle to measure area of photocell. (If area is known from manufacturer's specifications, this step can be omitted.)

2. Connect photocell, milliammeter A, and resistor R_L as shown (Figure 16).

3. Point active area of photocell directly at sun; record value of current flowing through meter.

4. Calculate electrical energy (E_{out}) produced per unit time (power in watts). Use $E_{out} = I^2R_L$, where I is measured current (in amperes) and R_L is resistance (in ohms).

5. Calculate optical energy per unit time (obtained from the known power per unit area delivered by sunlight to the earth's surface = 0.11 joule/s/cm²); total energy per unit time (power in watts) is 0.11 x area of photocell (in cm²).

6. Calculate efficiency η of energy convertor: η = electrical energy per unit time ÷ optical energy per unit time. η = ________

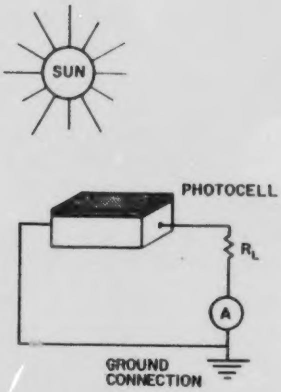


FIG. 16

SELF CHECK

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1. Describe the concept of energy convertors as it applies to all energy-conversion devices.

2. Identify and describe the operation of the energy convertor used to make the following conversions of energy:

   a. Electrical energy to thermal energy.
   b. Thermal energy to fluidal energy of steam.
   c. Optical energy to thermal energy.
   d. Fluidal energy to mechanical rotational energy.
   e. Electrical energy to mechanical energy.
   f. Mechanical energy to fluidal energy.
   g. Mechanical energy to electrical energy.
   h. Thermal energy to mechanical energy.

3. A 10-hp electric motor operates with an efficiency of 82%. Determine its electrical input power in kilowatts. (1 hp = 746 W.)

4. A solar-cell array received 750 J of radiant energy in 3 seconds and operates with an efficiency of 8.7%. Determine the average output power during this time interval.

5. A car with a mass of 1000 kg is raised through a height of 2 m by a hydraulic lift. The energy used originated as burning coal. The following energy convertors were used in the delivery of the necessary energy:

   Boiler — 75%
   Turbine — 82%
   Generator — 62%
   Electric motor — 79%
   Fluid pump — 93%
   Hydraulic cylinder — 96%

   Determine the overall efficiency of the system and the mass of the coal that must be burned initially to lift the car, if the energy content of the coal is 13,500 Btu/lb.

6. A heater has a thermal power of 4000 watts. If the heater operates on 220 volts, what current does the heater draw?

7. Derive an expression which gives the thermal power dissipated in a resistive element in terms of the current through the element and the element's resistance.

8. Calculate the efficiency of an energy converter that uses 400 joules of energy in 3 seconds and produces 1000 joules of energy in 5 seconds.

9. How long will it take a heater to produce 20,000 joules of heat energy if the heater operates on 110 volts and draws a current of 12 Amps? Assume the heater is 90% efficient.

10. How much could the heater of problem 9 raise the temperature of 1 kg of water in 1 minute? (Recall that $H = mc\Delta T$.)

11. How many 30Ω resistors in series are necessary to dissipate at least 4000 J/s of heat energy if the current through the resistors is 2 Amps? (Hint: Resistances in series add.)

12. A lever and pulley system designed to lift heavy objects is 80% efficient. If a man does 300 joules of work each second to lift a 400 kg object, how long will it take the man to lift the object through a vertical height of 2 meters?

13. A 100 kg miniature motorized vehicle has an overall efficiency of 11%. That is 11% of the energy created by the burning of the motor fuel is translated into the motion of the vehicle. What speed can the vehicle attain in 20 seconds, starting from rest, if the burning of the motor fuel liberates 1000 joules of energy each second?

14. How many calories are used up when a man lifts a 50 kg mass through a vertical height of 1 meter if the process of lifting the mass is 10% efficient?

15. What is the operating voltage of a toaster which is 80% efficient, draws a current of 6 Amps, and has a power rating of 400 watts?

16. What is the efficiency of a process which converts electrical energy into stored gravitation energy of a 3 kg block, if the operating voltage and current are 110 volts and 100 mA and the motor can raise the block 1 meter in 6 seconds?

17. How much water is in a calorimeter if when a heater with an output power of 200 watts is placed in the calorimeter, the temperature of the water increase 3 C°/second? Assume all of the heat produced by the heater goes into heating the water.

18. How much power would be required to maintain the speed of a 3000 kg car at 40 m/s if the car was 100% efficient?

19. An inclined plane can be thought of as a mechanism to convert the gravitational energy of an object into kinetic energy. What is the "efficiency" of a 6 meter long plane inclined at 20° if a 5 kg sperical object attains a speed of 6 m/s by rolling from the top to the bottom of the inclined plane?

20. A certain conductor ceases to conduct at a temperature of 1000°C. If the resistance of the conductor is 6Ω, find the maximum current the conductor can withstand if the mass of the conductor is 120 grams and its specific heat is 0.9 cal/gC°. Assume the conductor is originally at 20°C, the current is to flow for 10 minutes and the conductor is thermally insulated from its surroundings.

21. How high can a 50% efficient energy converter lift a 6 kg mass in 6 seconds if the input power is 100 watts?

22. The efficiency of a certain windmill can be modeled as a function of blade length only. Assume the efficiency as a function of blade length is, $\eta = \ell n(L)/L^2(200)$, where L is the blade length. Determine graphically the optimum blade length. (L is in meters and cannot be less than 1 meter.)

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TRANSDUCERS

CHAPTER 10-0

The operation of all technical systems requires that information about those systems be made available to some controlling mechanism. An automobile, for example, may contain gages that indicate speed, fuel level, oil pressure, and temperature. Each of these gages is connected to a device called a "transducer." The transducer converts information about the conditions in the automobile into a signal transmitted to the gage, where it is displayed.

Transducers, then, are devices that accept an input signal containing information in one energy system (engine temperature, for example) and convert it into an output signal containing the same information in another energy system (displacement of a needle on a calibrated scale).

This module discusses transducers commonly used to transform information about mechanical, fluidal, electrical, thermal, and optical systems into an electrical or a mechanical signal.

OBJECTIVES

Upon completion of this module, the student should be able to:

- In a short paragraph, define a transducer. Include a distinction between those transducers that require external energy sources and those that do not.
- Describe the operation of the following transducers:
    - Strain gage.
    - Accelerometer.
    - Microphone.
    - Turbine flow meter.
    - Barometer.
    - Meter movement.
    - Thermocouple.
    - Thermistor.
    - Bimetallic strip.
    - Photoconductive cell.
    - Photovoltaic cell.

DISCUSSION

To "transduce" means "to lead across." The function of a transducer is to move an information-carrying signal from one energy system to another. Transducers are called "sensors," because they provide sensory input to a monitoring or to a control system, just as human senses provide information to the brain.

Any device that transforms a signal (as described above) can be called a "transducer." Many types of transducers (electrical, mechanical, fluid, and thermal) operate in all the energy systems discussed in previous modules in this series. For each type, transducer function remains the same: to transform information from one system form to another.

Transducers can be regarded as having an input and an output (Figure 1). The transducer (shown as a box) accepts some input quantity, which may represent a physical property, and produces an output, which is the response of the transducer to that property.

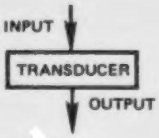


FIG. 1

An electrical input (Figure 2) and a mechanical output illustrates this principle more specifically. In this example, input is an electrical current; the transducer is an ammeter; and the output is the mechanical deflection of the needle on the meter.

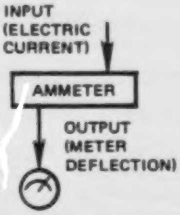


FIG. 2

Output usually varies with input, thus giving a transducer a capability as a measuring device. For example, voltage (or current) output from a photocell is indicative of the amount of light energy incident upon the photocell. Thus, a photocell may be used as a quantitative detector of light energy.

Another aspect of a transducer is that it must be capable of transmitting, as well as transforming, a signal. A transducer often operates on waves or currents (e.g., light waves, acoustic waves, or electric currents). Again, the photocell provides an excellent example: it receives light waves and emits electrical currents.

Some transducers actually are energy convertors; that is, they convert the energy of the input signal directly into energy of the output signal. The photocell described above is an example. When light strikes the photocell, some of the light energy is converted directly into electrical energy; therefore, any of the energy convertors discussed in Concept Module 9-0, "Energy Convertors," can be used as transducers. One advantage of this type of transducer is that it requires no external energy source for operation.

Other transducers do require an external energy source of their own. In most cases, this source is in the form of an electrical current through an element, whose resistance changes in response to some physical quantity. This type of transducer does not convert one type of energy directly into another; it changes its resistance in response to the input energy and, thus, changes the rate of flow of some quantity through itself. Most such transducers operate on electrical power, but a few devices using fluid flow as an external energy source also are used in some fluidal-control systems.

The remainder of this module discusses a variety of transducers. They are classified according to the energy systems in which the input signals originate. In all cases, the output signal is either electrical (current or voltage) or mechanical (position of an indicator).

MECHANICAL TRANSDUCERS

MICROPHONE

The most common device for changing a mechanical vibration wave to an electrical wave is the microphone.

Different types of microphones employ different physical principles of operation. One widely-used type is the carbon microphone, which contains a diaphragm that moves (vibrates) in response to acoustic waves incident upon it (Figure 3). Immediately behind (and in contact with) the diaphragm is a small cup filled with tiny grains of carbon. A low-voltage electrical current passes through these carbon grains. When the diaphragm moves in response to sound waves pressing on it, electrical resistance in the carbon medium changes, and electric current through the carbon varies accordingly. If the sound is intense, pressure on the diaphragm increases, pressing the carbon grains together more tightly.

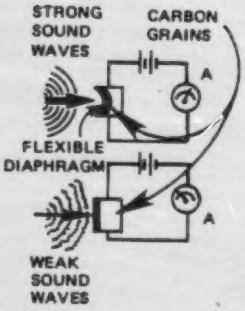


FIG. 3

Because highly-compressed carbon offers less resistance than loosely-packed carbon, the amount of electricity flowing through them increases. If sound is weak, only a light pressure is incident upon the diaphragm, and the grains are loosely-compacted; therefore, resistance is high, and less electric current flows through the carbon.

The diaphragm and the carbon grains form a transducer, changing acoustic waves into varying electrical current. And, although such a carbon microphone does not exhibit high fidelity, it does represent a good example of a transformer for sound energy into electrical energy. (Other, more sophisticated, microphone types convert sound more faithfully.)

PIEZOELECTRIC PRESSURE TRANSDUCER

Some types of crystalline material exhibit a phenomenon called "piezoelectric effect." When mechanical pressure is applied to certain crystal types, they are strained, or slightly deformed. This strain causes an electric charge to appear on certain surfaces of the crystals (Figure 4).

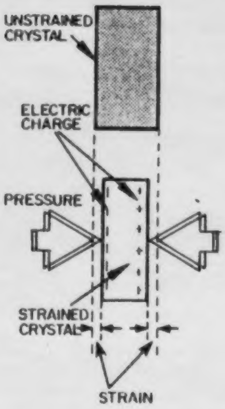


FIG. 4

Application of pressure results in strain, causing electrical charge of opposite polarity to appear on the two surfaces. (In reality, a complicated relationship exists between the direction of the applied force, the crystalline structure, and the surfaces on which the charge appears. Because this relationship is beyond the scope of this module, a simplified version is presented.)

When pressure is released, the charge disappears rapidly. Thus, a piezoelectric crystal must be considered a transducer of mechanical energy into electrical energy. The charge on the surface leads to a voltage in an electric circuit connected to the crystal. Thus, the crystal can be used as a pressure sensor. Quantitative measurement can be made of the pressure if the voltage is measured.

A piezoelectric crystal also can serve, in reverse, as a transducer of electrical energy into mechanical energy; that is, voltage applied to a piezoelectric crystal causes it to undergo deformation and to exert mechanical pressure.

The piezoelectric effect is observed only in certain types of crystals; for example, in Rochelle salt (sodium potassium tartrate) and in tourmaline (a semiprecious gem). Significant properties of several piezoelectric crystals, summarized in Table 1, include voltage that can be obtained in an open electrical circuit for unit applied pressure and for unit thickness.

TABLE 1. PIEZOELECTRIC CRYSTAL PROPERTIES

Crystal	Voltage Per Unit Thickness Per Unit Applied Pressure $\frac{\text{volt} \times \text{meter}}{\text{newton}}$	Coupling Coefficient k (Efficiency)
Quartz	0.055	9.9%
Rochelle salt (at 30°C)	0.098	78%
Ammonium dihydrogen phosphate	0.178	25%
Ethylene diamine tartrate	0.152	21.5%
Lithium sulfate	0.165	35%
Tourmaline	0.0275	9.2%

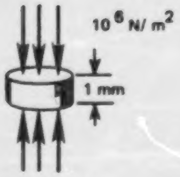
The coupling coefficients (Table 1) are efficiencies for conversion of electrical energy into mechanical energy. Values listed are obtainable only with careful design; but, as Table 1 shows, values can be quite high. Thus, under certain conditions, piezoelectric transducers can be efficient convertors of mechanical energy into electrical energy.

Piezoelectric crystals have many applications, but usually are employed in dynamic conditions, in which there are changing pressures or accelerations. Under steady conditions, when a static force is applied, the charge gradually leaks off the surface. Piezoelectric crystals can be found, therefore, in dynamic devices, such as phonograph pickups, high-quality microphones, accelerometers, and ultrasonic submarine detectors.

EXAMPLE A: CALCULATION OF PIEZOELECTRIC VOLTAGE

Given: A pressure of 10^6 newton/meter^2 is applied to a piezoelectric crystal of ammonium dihydrogen phosphate 1 mm thick.

Find: Resulting open circuit voltage.



Solution: From Table 1, voltage per thickness per unit applied pressure is given for ammonium dihydrogen phosphate as 0.178 volt x meter/newton. Voltage is given by

$$V = kph$$

where

k = coupling coefficient.
p = pressure on crystal.
h = height (thickness).

Therefore —

$$V = \left(0.178\ \frac{\text{volt x meter}}{\text{newton}}\right)\left(10^6\ \frac{\text{newton}}{\text{meter}^2}\right)\left(0.001\ \text{meter}\right)$$

$$V = 178\ \text{volts}$$

STRAIN GAGES

Strain gages are used to measure dimensional changes in a sample of material. A strain gage consists of a resistive element whose resistance changes in response to the strain of the element. The resistance of a wire was described in Concept Module 5-0, "Resistance," by Equation 1.

$$R = \frac{\rho \ell}{A} \qquad (1)$$

where

R = resistance.
ρ = resistivity of material.
ℓ = length of conductor.
A = cross-sectional area of conductor.

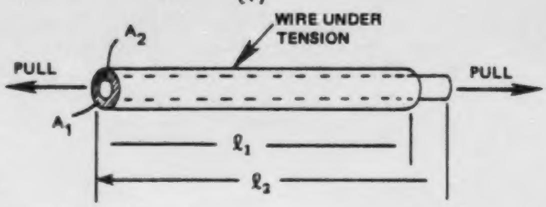


FIG. 5

If the shape of the conductor changes, as shown in Figure 5, then both A and ℓ change, producing a change in the resistance of the conductor. The strain gage is connected to a constant-voltage source; thus, a change in its resistance causes a change in the current through the gage. This change in current is proportional to the strain of the material to which the strain gage is bonded.

OTHER MECHANICAL TRANSDUCERS

Other mechanical transducers include phonograph needles and cartridges and position sensors, in which a position of an object is transformed into an electrical signal by a change in the value of a variable resistor. In rotational systems, mechanical rotational energy often is converted to electrical energy by a generator. Examples of this appear as an intermediate step in some of the fluidal transducers discussed in the following section.

FLUIDAL TRANSDUCERS

TURBINE (FLUID) FLOWMETER

"Turbine flowmeters" are devices inserted into a pipe or tube to measure the rate of fluid flow. A multi-bladed turbine (Figure 6) rotates in proportion to the velocity of the fluid flow past its blades. A coil of wire is placed next to the outside of the pipe. As the magnet rotates, it generates a short pulse of current in the coil.

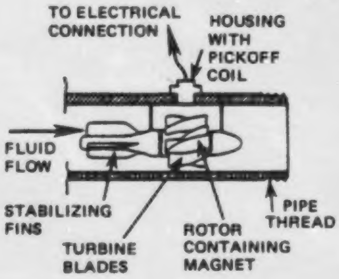


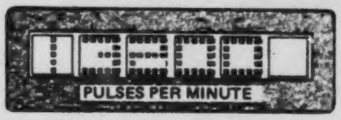
FIG. 6

This method of sensing relies upon electromagnetic induction, a phenomenon discovered by Michael Faraday in 1831. Motion of the magnet induces a voltage, therefore a flow of current, in a nearby coil of wire.

Because of electromagnetic induction, fluid flow through the pipe generates an electrical pulse at a rate proportional to the flow. Each pulse represents passage of a known volume of fluid past the rotor. The cumulative pulses can be fed directly into a digital display that indicates total flow. Also, the pulses can be used to drive an electrical meter to indicate flow rate. Such devices are important in industry for measuring, recording, and controlling flow of chemicals, petroleum, cryogenic liquids, and the like. The following example shows how a known calibration can be converted into a flow rate.

EXAMPLE B: CALCULATION OF FLOW BY FLOWMETER

Given: A commercial turbine flowmeter is calibrated (by number of pulses registered by pickup coil per unit volume of liquid passing through the turbine) at 1200 pulses per gallon and registers 13,200 pulses per minute.



Find:
(1) Flow rate.
(2) Flow (in gallons) in 6-minute period.

Solution: 13,200 pulses/minute x 6 minutes = 79,200 pulses

Total flow = 79,200 pulses ÷ 1200 pulses/gallon
= 66 gallons in 6 minutes

Flow rate = 66 gallons/minute/6 minutes
= 11 gallons/minute

This same principle is used in a device called an "annemometer," used to measure wind velocity.

PRESSURE-SENSING DEVICES

Figure 7 displays the parts of a barometer used to measure atmospheric pressure. The aneroid capsule is a flexible, sealed metal can. When the external pressure changes, the height of the capsule changes. This displacement is converted into a needle deflection by a gear drive. In some cases, the gear drive is connected to a variable resistor to transform the output into an electrical signal.

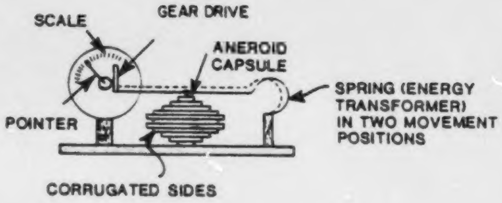

SCALE
GEAR DRIVE
ANEROID CAPSULE
SPRING (ENERGY TRANSFORMER) IN TWO MOVEMENT POSITIONS
POINTER
CORRUGATED SIDES

FIG. 7

The Bourdon tube (Figure 8) also transforms a pressure change into a needle deflection. As pressure in the tube increases, the free end moves upward; a decrease in pressure causes a downward movement.

ELECTRICAL TRANSDUCERS

METER MOVEMENT

Figure 9 illustrates the construction of a meter movement that transforms the magnitude of an electrical current into the deflection of a needle on a scale. The meter consists of four basic parts: (1) A permanent magnet provides a constant magnetic field. (2) A coil of wire is wrapped around (3) an iron cylinder suspended in this field. (4) A coil spring is arranged to hold the needle of the meter at zero when no current passes through the coil. When a current flows through the coil, a magnetic field is produced at an angle to the field of the permanent magnet. The magnetic forces produced rotate the movable coil and cylinder until the torque applied by the spring balances that applied by the magnetic field.

The scale of the meter can be calibrated to provide measurement of any quantity proportional to the current through the meter coil. Such meter movements are used widely in all areas of technology.

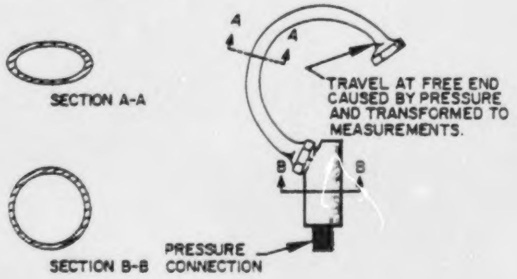

A
A
SECTION A-A
TRAVEL AT FREE END CAUSED BY PRESSURE AND TRANSFORMED TO MEASUREMENTS.
B
B
SECTION B-B
PRESSURE CONNECTION

FIG. 8

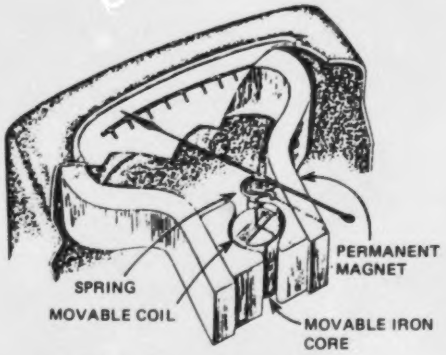

PERMANENT MAGNET
SPRING
MOVABLE COIL
MOVABLE IRON CORE

FIG. 9

THERMAL TRANSDUCERS

THERMOCOUPLE

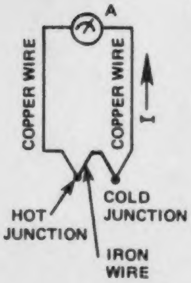


FIG. 10

A thermocouple, an extremely simple transducer, transforms thermal energy into electrical energy. Widely-used for temperature sensing, thermocouples depend upon the fact that electrical current flows in a circuit of two different metals (provided the metals are joined to form a complete circuit, and provided the two junctions are kept at different temperatures). In a thermocouple formed of copper and iron wires, if cold junction is kept at 0°C, and if the other junction is heated, current flows as indicated in Figure 10. Current flow shows that an electromotive force (voltage) exists between the two junctions. This voltage, produced by temperature difference, is typically small — on the order of a fraction of a millivolt for a temperature difference of 1°C.

Because voltage varies with temperature differences, thermocouple transducers make excellent temperature-measuring devices.

A third metal in the circuit does not change the voltage if its junctions are at the same temperature. Thus, a measuring device, such as a voltmeter or potentiometer, can be inserted into a circuit without affecting its voltage.

In practice, temperature of one junction is kept at a known, fixed temperature. This set temperature can be obtained, for example, if the cold junction is immersed in an ice-water bath at 0°C. Then the unknown temperature of the other junction can be derived from the measured voltage.

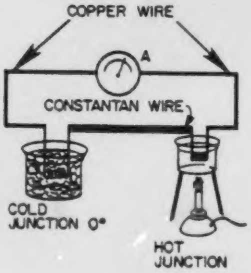
Approximate voltage values that can be expected from various pairs of metals when the cold-junction temperature is 0°C are shown in Table 2.

Example C makes use of data given in Table 2 and shows use of a thermocouple as a temperature-measuring device.

TABLE 2. APPROXIMATE VOLTAGE VALUES FOR CERTAIN THERMOCOUPLES (COLD JUNCTION AT 0°C)

Metals	Useful Temperature Range (C°)	Voltage (Millivolts/C°)
Iron-constantan (60% Cu - 40% Ni)	0 to 800	0.050
Chromel-alumel	300 to 1300	0.040
Platinum/rhodium-platinum	600 to 1500	0.009
Copper-constantan	-200 to 400	0.040

EXAMPLE C: THERMOCOUPLE JUNCTION TEMPERATURES



Given: A copper-constantan thermocouple, with its cold junction at 0°C, registers a voltage of 4.8 millivolts.

Find: Temperature of the other junction.

Solution: From Table 2, a temperature difference of 1 C° produces 0.04 millivolts. Thus, the temperature difference between the junctions is 4.80/0.04 = 120 C°. Because the cold junction is at 0 C°, the other junction is at 0 + 120 C° = 120 C°.

OTHER TEMPERATURE-MEASURING DEVICES

Several additional temperature-measuring devices are in common use. In an ordinary thermometer, the level of a liquid in a capillary tube indicates the temperature of the liquid and its surroundings. A thermistor is a resistive element whose resistance changes in response to its temperature. A bimetallic strip consists of two strips of different metals bonded together. As the temperature of the strip increases, one metal expands more rapidly than the other, causing the strip to bend. This type of thermal sensor is used widely in the thermostats controlling industrial and residential heating and cooling systems.

OPTICAL TRANSDUCERS

PHOTOCONDUCTIVE PHOTOCELL

A photoconductor is a transducer of the intensity of optical (light) energy into electrical voltage or current (Figure 11). The sensitive element in a photoconductor is a semiconductor, such as silicon. Bias voltage applied across a semiconductor causes electrical current to flow through a load resistor. Because the semiconductor typically has low electrical conductivity, current is small when a semiconductor is in the dark. But light falling upon the semiconductor releases free electrons within the semiconductor; electrical conductivity increases, causing current to increase and, according to Ohm's law ($V = IR_L$), voltage V to increase.

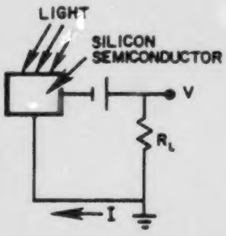


FIG. 11

Although the output appears primarily as a forcelike quantity (voltage), it may be used to drive current in an external circuit.

PHOTOVOLTAIC CELL

The photovoltaic cells described in Concept Module 9-0, "Energy Convertors," and mentioned earlier in this module also are used as optical-to-electrical transducers. Unlike photoconductive cells, photovoltaic cells produce their own voltage by a conversion of optical energy into electrical energy. They require no external voltage source when operated in the photovoltaic mode. These devices also can be connected in a biasing circuit similar to Figure 11 and operated in the photoconductive mode. This connection is made to improve both sensitivity and response time.

SUMMARY

A transducer is a device capable of accepting an input signal containing some information from one energy system and of producing an output signal containing the same information in another energy system. Some transducers are energy convertors that convert input energy to output energy. These transducers require no external energy source. Others change their resistance in response to the input and, thus, change the rate of flow of some quantity, usually of electric current. This second class of transducers requires an external energy source to supply the current to the transducer.

There always is a relationship of the output of a transducer to its input. For example, as temperature increases, voltage output of a thermocouple increases; as fluid flow increases, the counting rate of a turbine flowmeter increases. In other words, the output of a transducer gives a quantitative indication of its input. For this reason, transducers are used widely as measuring devices (in science, engineering, and industry) of diverse quantities (temperature, pressure, flow rate, illumination levels, and so on).

TEMPERATURE MEASUREMENT

MATERIALS

Commercial thermocouple (copper-constantan or equivalent).
Millivoltmeter.
Watch or clock.
Bunsen burner.
Ring stand and clamp.
Two beakers.
Water and ice.
Wiring and clips for electrical connections.
Graph paper.

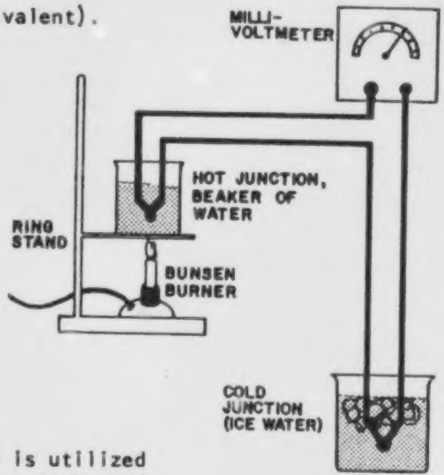


FIG. 12

PROCEDURE

In this experiment, a thermocouple transducer is utilized to measure temperature of water in a beaker as water is heated.

1. Set up apparatus (Figure 12). The cold junction contains mixture of water and ice at 0°C. The hot junction is in beaker of water heated by a bunsen burner. (Use one of thermocouples listed in Table 2.)
2. Record initial reading of millivoltmeter: ___________.
3. Light bunsen burner; begin heating water.
4. Record reading of millivoltmeter at one-minute intervals as water heats: ___________.
5. When water begins to boil, record final reading of millivoltmeter; turn off bunsen burner: ___________.
6. Calculate water temperature for every recorded value of voltage; use value from Table 2 for response of particular thermocouple.
7. On graph paper, record temperature versus time; compare final value with known boiling temperature of water.

CONVERSION EFFICIENCY

MATERIALS

Electrically powered horn.
Battery or d.c. voltage supply.
Switch.
Ammeter.
Commercial piezoelectric microphone pressure transducer.
Oscilloscope.
Wiring for electrical connections.
Meter stick.

PROCEDURE

This experiment requires two transducers — a horn (electrical-to-mechanical) and a piezoelectric pressure sensor (mechanical-to-electrical). Measure efficiency of conversion of electrical energy into acoustic energy.

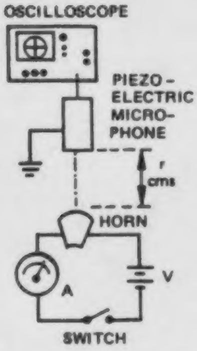


FIG. 13

1. Connect apparatus (Figure 13). The piezoelectric transducer (microphone) can be about 1/3 meter from horn.

2. With meter stick, measure distance r (in cm) from horn to piezoelectric transducer microphone. Record here: __________.

3. Close switch to activate horn; measure current I (through horn) with ammeter A. Record here: __________.

4. Measure voltage produced by piezoelectric transducer. (Use oscilloscope.) Record voltage: __________.

5. Open switch to turn off horn.

6. Calculate pressure p at piezoelectric transducer; use manufacturer's calibration of voltage versus pressure.

7. Calculate energy per unit time in sound wave:

$$\text{Energy per unit time} = 4 \times 10^{-7}\ \pi r^2 p^2/\rho c$$

NOTE: In the above equation, energy per unit time is in joules per second, r is in cm, and p (from Step 6) is in dyne/cm^2. Use known values of $\rho = 0.00129$ g/cm^3 for density of air and $c = 3.32 \times 10^4$ cm/s for velocity of sound in air. The above formula is an approximation; it assumes that sound spreads equally in all directions from horn. (Because horn probably has directional properties, formula may not be strictly accurate, but is sufficient for this experiment.)

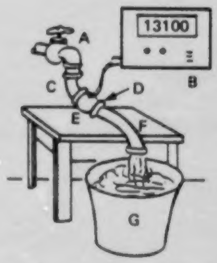
8. Calculate electrical energy (per unit time) delivered to horn. This value is given (in joules per second) by product of voltage V (volts) time current I (amps) from horn circuit.

9. Determine efficiency (η) of conversion of electrical energy into acoustic energy by dividing result of Step 7 by result of Step 8.

FLOW MEASUREMENT

MATERIALS

Turbine flowmeter — covering range of $\simeq$ 5 gal/min. (Model 4900-12 with model 4100 meter readout from Brooks Instrument Division, Emerson Electric Company, or equivalent).
Cold-water supply.
Tubing and connectors or hose clamps, as needed.
Stopwatch.
Pail (capacity of at least 5 gallons).



A - COLD WATER TAP
B - INDICATING UNIT
C - TUBE
D - HOSE CLAMPS
E - FLOW METER
F - TUBE
G - 10 - GALLON PAIL

FIG. 14

PROCEDURE

In this experiment, a turbine flowmeter will be employed as a transducer to measure flow of water.

1. Connect flowmeter to cold water tap; use rubber tubing and clamps as needed (Figure 14); connect indicating unit to flowmeter; direct output of final length of tubing into the pail; mark 5-gallon capacity on inside of pail.

2. Quickly turn on cold water; simultaneously start stopwatch.

3. Observe and record reading of flowmeter indicator: _________.

4. When water reaches 5-gallon mark, stop stopwatch and record reading of flowmeter indicated: _________.

5. Calculate flow rate (gal/min): 5 gallons divided by time (in minutes) to fill pail: _________.

6. Compare results (from Step 5) with flow measured in Steps 2 to 4.

7. Discuss differences, if any.

SELF CHECK

1. Describe the function of a transducer.

2. Describe the differences between transducers that convert input energy directly into output energy and those that do not. Give an example of each type in the following energy systems:

   Mechanical.
   Fluidal.
   Thermal.
   Optical.

3. Describe the operation of the following transducers:

   Meter movement.
   Microphone.
   Accelerometer.
   Barometer.
   Thermocouple.
   Photovoltaic cell.
   Bimetallic strip.
   Strain gage.
   Turbine flow meter.
   Thermistor.
   Photoconductive cell.

4. What pressure must be applied to piezoelectric crystal of lithium sulfate to produce an open circuit voltage of 200 volts if the crystal is 5 mm thick?

5. How much electrical energy can a quartz crystal put out if 1000 joules of mechanical energy is applied to the crystal?

6. If the open circuit voltage resulting from an applied pressure of 10^7 N/m^2 is 1960 V and the piezoelectric crystal is 2 mm thick what material is the crystal probably made of? (See Table 1.)

7. What is the resistivity of a conductor if a 5 meter length of the conductor has a resistance of 10 Ω? The cross-sectional area of the conductor is 0.01 m^2.

8. A wire is placed under a strain of 1 N/m^2 and the resulting change in the current through the wire is 1 mA. Find the strain necessary to produce a change of 10 mA in the current through the wire.

9. What is the flow rate through a pipe if a flowmeter is calibrated at 1500 pulses per gallon and registers 60,000 pulses in a 10 minute period?

10. A flow meter is to be calibrated. If a flow rate of 2 gal/min produces 25,000 pulses in 5 minutes, how many pulses correspond to one gallon?

11. A chromel-aluminum thermocouple registers a voltage of 20 millivolts. If the cold junction is at 0°C what is the temperature of the other junction?

12. An iron-constantan thermocouple is to be calibrated. What voltage should the thermocouple register if the temperature of the hot junction is 600°C and that of the cold junction is 0°C?

13. What is the approximate power carried in a sound wave if $r = 0.2$ cm and $p = 10^6$ dynes/cm^2. (Use the formula given in the laboratory section.)

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VIBRATIONS AND WAVES

CHAPTER 11-0

Energy can be transferred from one place to another in a variety of ways. A bowler, for example, transfers kinetic energy of the rolling ball to the bowling pins. Here, one object, the ball, interacts with another, the pin. In other instances, energy moves in the form of waves, without the transfer of bulk matter. Sound, for instance, is a wave that carries energy from a source (megaphone) to a receiver (ear). Light is a wave that carries energy from a source, such as the sun, to a receiver, such as the eye. Wave motion always is characterized by a vibration or displacement at some location (source) and is propagated through a medium (as sound through air) or through a vacuum (as light through space). In some wave motion, the vibration is along the direction of propagation and is described as "longitudinal wave motion." In other instances, the vibration is perpendicular to the direction of propagation and is described as a "transverse wave motion." Sound energy propagates as a longitudinal wave, and light energy propagates as a transverse wave. This module examines wave motion in general and discusses common examples of vibrations and waves.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Distinguish between longitudinal and transverse waves by giving at least two examples of each type and by drawing and labeling a sketch of each.

- Define the following words associated with waves and wave motion:

  Propagating medium.
  Wavelength.
  Frequency.
  Period.
  Displacement.
  Amplitude.
  Phase.
  Standing wave.
  Constructive interference.
  Destructive interference.
  Beats.

- Calculate the wavelength of a wave, given its velocity and frequency.

- Interpret this equation, explaining the significance of each symbol:

$$y = A \sin 2\pi \left(\frac{x}{\lambda} - ft\right).$$

- Explain the meaning of the expression "the current leads the voltage by a given phase angle" by using sine-wave sketches of both current and voltage.

- Describe the superposition principle.

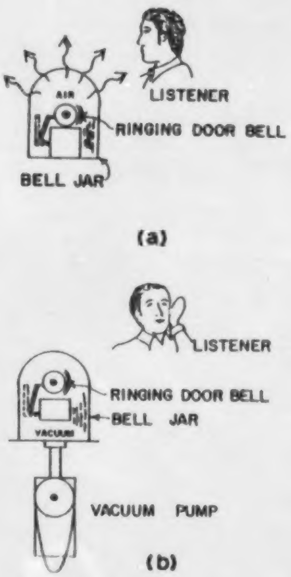
- Describe wave phenomena in each of the following energy systems:

  Mechanical.
  Fluidal.
  Electrical.

DISCUSSION

A wave is a disturbance in which energy is transferred from one location to another. Some well-known examples of waves are sound waves that result from vibrating guitar strings, seismic waves that move through the earth following an earthquake, water waves produced by the vertical motions of the surface of a lake, and light waves that can travel through a vacuum.

Several types of waves require a material in which to travel, or propagate, themselves. This material is referred to as a "medium." Sound is an example of a wave that requires a medium through which to move. The propagation of sound waves through air is a fact of common experience. Sound also can travel through liquids and solids. Sound cannot travel through a vacuum (Figure 1).



(a)

(b)

FIG. 1

Electromagnetic waves do not require a medium. Light, for example, is an electromagnetic wave that reaches the earth from the sun through the vacuum of outer space. Other waves of this type are radio waves, X-rays, and heat waves.

The energy transmitted by a wave may be in the form of a single burst, or pulse, or it may be in the form of many identical pulses, evenly spaced. Consider a person holding one end of a rope fastened to a wall on the other end (Figure 2). The person makes one up-and-down motion with an arm. This motion is transmitted to the left end of the rope, which then transmits the motion gradually down the entire rope length. The result is the generation of a single pulse wave, which moves down the rope (Figure 2).

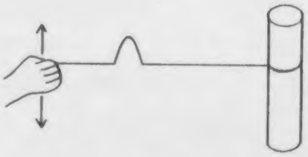


FIG. 2

When the person moves an arm up and down repeatedly, the wave produced is made up of a series of identical pulses (Figure 3). When wave pulses are repeated at regular intervals, the wave is called a "periodic wave."

A shock wave moving away from the scene of an explosion is an example of sound energy being transferred in the form of a single pulse through the air. A musical tone coming from a vibrating guitar string is an example of a continuous wave of sound energy.

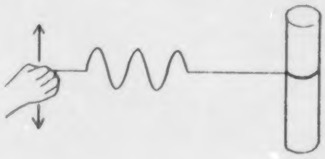


FIG. 3

TRANSVERSE WAVES

"Transverse" waves are those in which the disturbance producing the wave moves perpendicularly to the direction in which the disturbance is propagated. The wave motion produced in a rope (Figure 3) is one example.

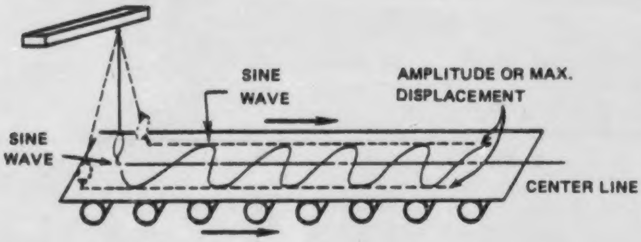


FIG. 4

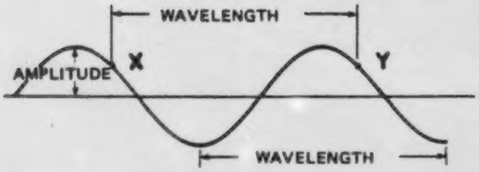


FIG. 5

Here, a sine wave will be used as an example of transverse wave motion. Figure 4 shows a sine wave being produced by a swinging pendulum to which a pencil is fastened. A piece of paper is pulled under the pendulum at a constant rate of speed, while the pencil traces the path of the swinging pendulum. Motion that generates a sine wave (Figure 4) is called "simple harmonic motion."

Displacement at any time is the distance of the pendulum from its center, or equilibrium, position (Figure 4). The maximum displacement of the pendulum is called the "amplitude" of the wave. The energy transferred by the wave turns out to be proportional to the square of the amplitude. The wavelength λ is the distance between identical points of the wave — from crest to crest, for example. Both amplitude and wavelength are illustrated in Figure 5.

The pulses that make up periodic waves can have many shapes. Several pulse shapes are illustrated in Figure 6.

The frequency is the number of complete waves produced each second. When one wave is produced each second, the wave has a frequency of one hertz (Hz).

The time required for one complete wave pulse is known as the "period" of the wave. The period and the frequency are reciprocals of each other, as indicated by Equation 1.

$$T = \frac{1}{f}$$

or

$$f = \frac{1}{T} \qquad (1)$$

where

T = period of wave.
f = frequency of wave.

The position and motion of a given point on a wave defines the phase of the wave. Points that have the same displacement and that are moving in the same direction are in phase with each other. Thus, points x and y on the wave (Figure 5) are in phase.

LONGITUDINAL WAVES

A "longitudinal wave" is one in which the disturbance producing the pulses moves parallel to, or along the direction in which, the disturbance is propagated. If a slightly-stretched spring were attached to a barely-swinging pendulum in such a way that the pendulum would swing parallel to the spring (Figure 7), a longitudinal wave would be produced.

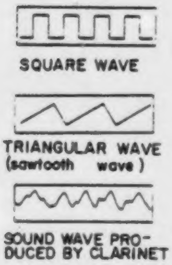

SQUARE WAVE
TRIANGULAR WAVE (sawtooth wave)
SOUND WAVE PRODUCED BY CLARINET

FIG. 8

As illustrated in Figure 7, when the pendulum has moved to its maximum right-hand position, a compression of the springs is produced. Similarly, when it has moved to the left-hand position, an elongation, or "rarefaction," is produced. This series of compressions and rarefactions produced represents a periodic longitudinal wave moving from left to right in the spring. Sound waves are of this type.

A comparison can be drawn between the compression and rarefactions of a longitudinal wave and the crests and troughs of a transverse wave (Figure 8).

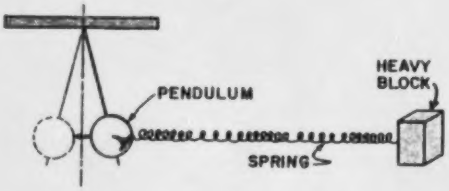


FIG 7

The definitions for amplitude, wavelength, frequency, and period presented earlier for transverse waves apply equally well to longitudinal waves. For example, the wavelength λ is the distance between adjacent compressions.

WAVE VELOCITY

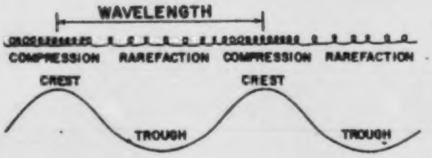


FIG. 8

A wave pulse, or disturbance, moves a distance equal to its wavelength in a time equal to the period of the wave. Since velocity is distance divided by time, the relationship expressed by Equation 2 holds true for all wave motion.

$$v = \frac{\lambda}{T} \qquad (2)$$

where
v = wave velocity.
λ = wavelength.

But since $T = \frac{1}{f}$, the value obtained is $v = \frac{\lambda}{1/f}$, or —

$$v = f\lambda \qquad (3)$$

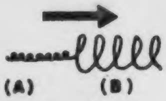


FIG. 9

The speed of a wave is affected greatly by the medium thorugh which the wave travels. For example, the sound of a moving train travels faster through the rails of the track than through air. Even in air, the speed of sound depends upon temperature and pressure of air. Other characteristics of a medium that can affect the speed of sound or light waves through that medium include density, elasticity, and index of refraction.

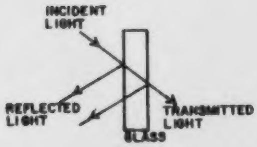


FIG. 10

Waves traveling through a uniform medium do not change velocity, but waves that travel from one medium to another medium do experience velocity changes. This can be illustrated by the longitudinal wave motion in springs. Consider a wave moving in a moderately-stiff spring (Figure 9). When the wave pulse from the moderately-stiff spring reaches the boundary between the two springs, some of the energy in the pulse will be transferred into the very stiff spring, where it will continue to travel at a slightly different velocity. The remainder of energy will be reflected by the boundary and will bounce back in the moderately-stiff spring. Thus, some of the energy is reflected, and some is transmitted. The same effect occurs, but in different proportions, if the wave travels from the stiff spring into a very soft spring.

A similar effect can be seen when a beam of light waves is directed on a flat piece of glass (Figure 10).

When a light wave strikes a flat piece of glass, some of the light enters the glass, and some is reflected. Again, some of the light passing through glass is reflected when light strikes the rear surface of the glass, and some exits the glass and passes into the air. The amounts of light reflected and transmitted will depend, among other things, upon the angle of incidence. Notice that reflection and change of velocity occur when the medium changes, first from air-to-glass, and then from glass-to-air.

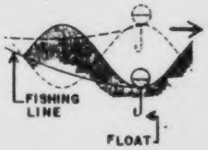


FIG. 11

THE WAVE EQUATION

Consider the motion of a single point of the medium as a wave passes that point. The motion of a float on a fishing line because of a water wave (Figure 11) is a good example. The float, moving more-or-less vertically up and down, is shown riding the valley and, later (dotted line), riding the crest of the wave. As the wave disturbance moves past the float, it undergoes a vertical displacement shown by the arrow (Figure 12). Notice that the motion of the float is perpendicular (up-and-down) to the direction in which the wave moves (left-to-right). The wave shown is necessarily, then, a transverse wave.

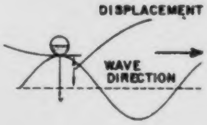
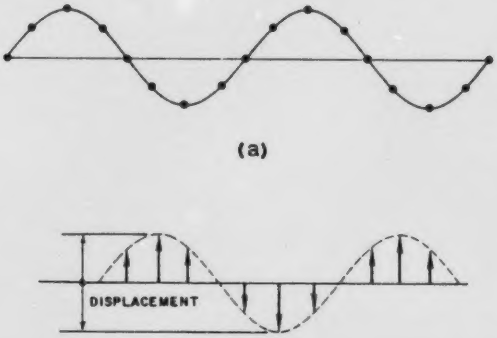


FIG. 12

The displacement of each point along a wave can be analyzed in the same manner. If a picture is taken of a moving wave, the photograph will show that each point along the wave has a different displacement. At the instant the photograph is taken, some of the points will be displaced (Figure 13a). However, each point will be seen to oscillate vertically with a periodic motion as time passes.

The momentary displacement of each point on the wave of Figure 13a is indicated by the arrows in Figure 13b. The direction of the arrow is the same as the direction of the displacement, and the length of the arrow indicates the amount of displacement of the point.

A mathematical analysis of wave motion leads to the general equation (Equation 4) for the displacement.



(a)

(b)

FIG. 13

$$y = A \sin 2\pi \left(\frac{x}{\lambda} - ft\right) \qquad (4)$$

where
y = wave displacement.
A = wave amplitude.
f = frequency of wave.
t = time.
x = distance along wave.
λ = wavelength.

In this equation y is the vertical displacement of a point a distance x from the last positive-going zero crossing of the wave at a time t. This general form of the wave equation may be applied to all wave phenomena. If analysis is to be in temporal terms only, x is set equal to zero. For spatial analysis t is set equal to zero.

Often, a sine wave can be understood readily when it is compared to wave motion around a circle. This relationship is pictured in Figure 14. Each angle drawn on the reference circle is 30°, or $\pi/6$ radians. Each division on the sine wave axis also is 30°, or $\pi/6$ radians. Notice how the points on the reference circle correspond to points on the wave. Thus, point 1 on the wave corresponds to 0 radian; point 2 corresponds to $\pi/6$ radian; point 3, to $\pi/3$ radian; and so on. One complete rotation on the reference circle corresponds to one complete sine wave. Now the phase of a point on the wave can be related to an angle. For example, point 6 on the wave has a phase of $5/6\ \pi$, and point 9 has a phase of $4/3\ \pi$.

Commonly 60-hertz alternating current (a.c.) electrical power produced in the United States is an example of wave motion applied to electricity. The voltage and current in an a.c. circuit varies with time according to a sine wave. The relationships for voltage and current as a function of time are given in Equations 5 and 6, respectively:

$$V_t = V_{max} \sin (2\pi\ ft) \qquad (5)$$

where
V_t = instantaneous voltage at time t.
V_{max} = maximum voltage.

$$I_t = I_{max} \sin (2\pi\ ft) \qquad (6)$$

where
I_t = instantaneous current at time t.
I_{max} = maximum current.

Note that these equations are for the temporal variations of voltage and current at a specific point. Thus, "x" of Equation 4 has been chosen as zero and the distance term does not appear in Equations 5 and 6. Many authors show Equations 5 and 6 as $V_t = V_{max} \sin \omega t$ and $I_t = I_{max}\ \omega t$ where $\omega = 2\pi f$ = "angular frequency"). Since 2π radians represents a full circle, $2\pi f$ is the angular velocity at which the reference circle in Figure 14 is repeated.

In a.c. circuits that contain only pure resistance, the voltage and current values at any instant of time (t) are "in phase" (Figure 15). In other words, both voltage and current values peak and trough in step.

When an a.c. circuit contains inductance and capacitance, the voltage and current may not be in phase. The angular difference between voltage and current is called the "phase angle." An out-of-phase condition is displayed in Figure 16. The phase angle θ shown in Figure 16 is 45°, or $\pi/4$ radians. The current is leading the voltage by 45° in this example. The voltage can lead the current (Figure 17). The phase angle θ can vary from 0° to 360°.

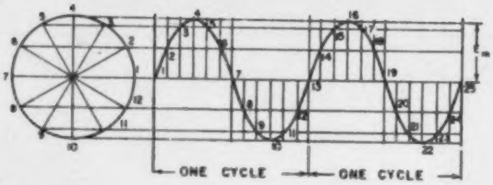


FIG. 14

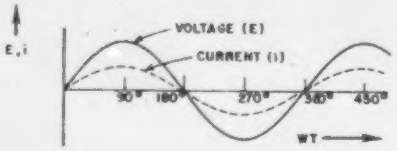


FIG. 15

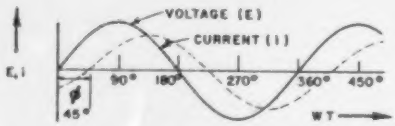


FIG. 16

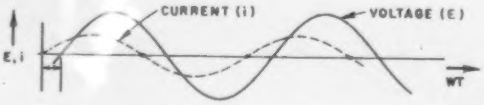


FIG. 17

SUPERPOSITION

When two or more waves traveling in the same medium overlap, a new wave is produced — a combination of the original waves. The "superposition principle" states that "the resultant wave will be the sum of the two components waves." Figure 18 illustrates the superposition of two waves. (The exact composite wave illustrated here only exists for the instant the component waves have the form shown.)

At position 1, displacement n belongs to wave 1, and displacement m belongs to wave 2. The two displacements are added to give the displacement m + n of the composite wave. At position 2, a displacement of p is produced by both waves, so that the composite displacement at position 2 is equal to p + p, as shown. In position 3, no displacement is produced by wave 2. The composite displacement, therefore, is equal to the displacement of wave 1. Position 4 shows that wave 2 has a negative displacement.

and that wave 1 has an equal positive displacement. Here the two displacements counteract each other to give zero net displacement. A similar analysis can be conducted at any number of desired positions to verify that the composite wave (Figure 18) is the result of the superposition of waves 1 and 2.

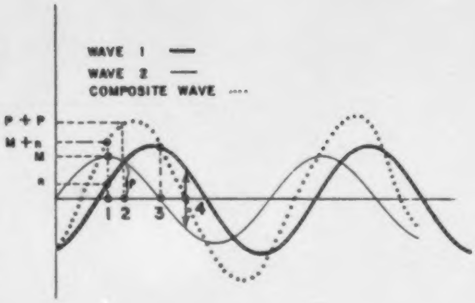


FIG. 18

STANDING WAVES

An important application of the superposition principle occurs in the formation of standing waves. A standing wave is produced when a train of reflected waves are superpositioned with an identical train of incident waves. The resultant wave will appear not to be traveling, and thus is called a "standing wave."

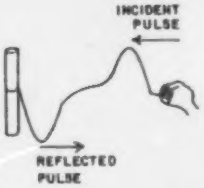


FIG. 19

A standing wave can be illustrated by a single pulse formed in a rope firmly attached at one end (Figure 19). A pulse is inverted if it reflects from the fixed end. When an incident pulse and an identical reflected pulse pass each other, their displacements cancel; therefore, zero displacement of the rope results during the time the pulses are in contact. After the pulses cross, they continue with their initial shape and direction.

When there are two continuous wave trains of the same wavelength and amplitude, but traveling in opposite directions, the composite wave appears to stand still. In this case, the two component waves do not add to zero everywhere at all times. Figure 20 shows the composite wave at four different instants.

The composite wave pattern shows points on the axis of the wave at which no displacement ever is observed. These points are called "nodes" and are one-half wavelength apart. The "antinodes" are points, also $\lambda/2$ apart, on the axis that correspond to maximum displacements of the standing wave.

Standing waves are produced in musical instruments, such as guitars or violins, where the disturbance generated by the musician travels backwards and forwards between the fixed ends of the wire. A pipe organ produces similar standing waves in pipes. The length and diameter of the pipes determine the characteristics of the standing waves and, thus, the characteristics of the music produced.

INTERFERENCE

Interference is the resultant cancellation or reinforcement that occurs when two or more waves are superposed. If two identical waves meet in phase (crest meeting crest and trough meeting trough), the principle of superposition predicts that a wave of twice the amplitude will result. Similarly, if the two waves meet out of phase by 180° ($\lambda/2$), then a crest of one wave meets a trough of the other wave, and zero amplitude results. Figure 21 illustrates two identical wave sources oscillating in step in such a manner as to project waves of equal wavelength and amplitude. The solid lines represent crests of the wave; the dotted lines correspond to troughs.

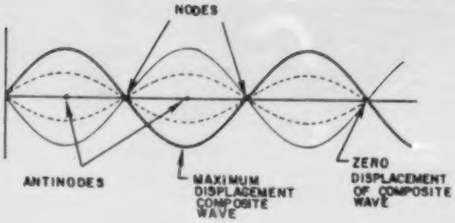

NODES
ANTINODES
MAXIMUM DISPLACEMENT COMPOSITE WAVE
ZERO DISPLACEMENT OF COMPOSITE WAVE

FIG. 20

At point 1, two troughs meet and produce a double amplitude. Point 2 is a position at which two crests meet and produce a double amplitude. Similarly, waves from the two sources meet in phase at points 3, 4, 5, 6, 7, and 8. These points are said to be points of "constructive interference." Thus, constructive interference occurs along the directions of OP, OP_2, and OP_2'. "Destructive interference" occurs at point 9, 10, 11, 12, 13, 14, 15, and 16, because a wave crest from one source meets a wave trough from the other source.

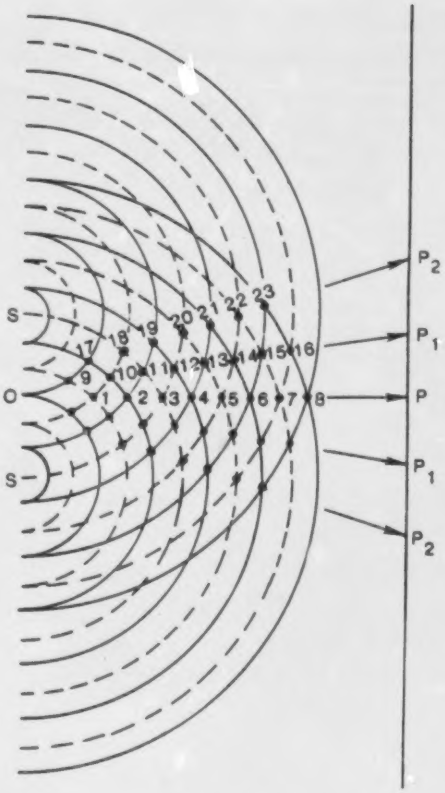


FIG. 21

Since a crest and trough are 180° out of phase, zero displacement results at these positions. Destructive interference occurs along the directions OP_1 and OP_1'.

Constructive and destructive interference is observed when two sources generate waves of equal wavelength or freuqency. If the sources produce waves of slightly different frequencies, a phenomenon known as "beats" will occur. Consider two slightly

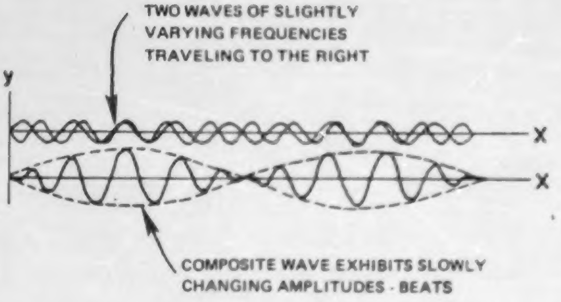

TWO WAVES OF SLIGHTLY VARYING FREQUENCIES TRAVELING TO THE RIGHT
y
X
X
COMPOSITE WAVE EXHIBITS SLOWLY CHANGING AMPLITUDES - BEATS

FIG. 22

different frequency waves traveling in the same direction (Figure 22). Addition of the two component waves of similar frequency is shown along the lower axis. Notice that where the component waves are in phase (or nearly so), the amplitude of the composite wave is large; and where the component waves are out of phase (or nearly so), the amplitude of the composite wave is small. The result shows two "pulses," or "beats." If the pulses are not too rapid, they can be detected separately by the human ear. Musicians listen for beats when tuning their instruments. The beat frequency turns out to be equal to the difference in frequency of the two component waves. Thus, if no beats occur, the instruments are in tune.

WAVE MOTION AS A UNIFYING CONCEPT

The basic characteristics of waves discussed in this module apply to all wave phenomena. The following are important examples of waves in several energy systems. The exception here is the thermal system, which exhibits no wave motion.

MECHANICAL SYSTEMS

In mechanical systems, both longitudinal and transverse waves are important. Longitudinal mechanical waves are compression waves in a material: Sound waves travel through a solid; compression waves are produced by a moving spring; and mechanical vibrations are present in machines and in structures. Transverse mechanical waves include the side-to-side motion that occurs in the strings of musical instruments and in torsional (twisting motion) waves. In mechanical structures, the reduction or elimination of wave motion often is a primary design consideration.

FLUIDAL SYSTEMS

The most important fluidal waves are compression waves (sound) in gases. The field of acoustics is based upon the study and control of these waves. The design of sound-delivery systems, for example, requires an analysis of the patterns of the radiated sound energy, the interference of sound waves from two or more sources, and the absorption and reflection of sound waves by materials.

ELECTRICAL WAVES

Electrical waves are among the most important to modern technology. Commercial electrical power is delivered as an electrical wave at a frequency of 60 Hz. Most electronic equipment performs its function, at least in part, through the manipulation of electrical waves. The concepts of superposition of waves and phase difference are particularly important in the analysis of electronic circuits.

X-rays, light, microwaves, and radio waves are examples of electromagnetic radiation. This type of transverse wave consists of oscillating electric and magnetic fields traveling through space; it is discussed in detail in Concept Module 13-0, "Radiation."

SUMMARY

A wave is a periodic disturbance in either a medium, such as air, or in a vacuum, such as space. Some waves require a medium; for example, sound waves require air through which to travel. Light waves and other electromagnetic waves do not require a medium. Wave energy can be transmitted in the form of a single pulse or in many evenly-spaced pulses.

Transverse waves are produced when the source moves perpendicularly (from side to side) to the direction in which the waves travel. Longitudinal waves are propagated when the individual parts of the source vibrate back and forth in the direction in which the waves travel.

Wave velocity is affected by the material through which waves travel. Two or more waves are superimposed when they overlap within the same medium to produce standing waves or interference.

Wave motion applies, as a unifying concept, to mechanical, fluidal, and electrical systems.

WAVES WITH OR WITHOUT A MEDIUM

MATERIALS

Bell jar.
Vacuum pump.
Door bell.
Dry cell.
Sound level meter (optional).
Flashlight or small laser.
Light meter (optional).

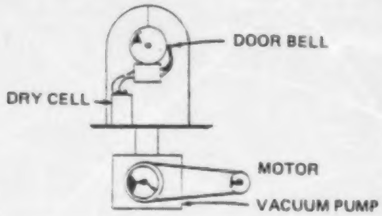


FIG. 23

PROCEDURE

1. Arrange door bell buzzer to operate under bell jar in place on vacuum pump. Figure 23 illustrates this experimental arrangement. Suspend buzzer inside bell jar or cushion it in such a manner that sound cannot be carried by conduction through solid material. Observe sound level of buzzer before air is removed from bell jar, and compare this level to sound level after air is removed. If sound level meter is available, you may use it to obtain more accurate comparisons. How well does sound travel through a vacuum?

2. Shine beam of light from flashlight through glass bell jar. Use light meter to compare amount of light transmitted through air with amount transmitted through vacuum. Does light require a medium for transmission?

WAVES IN A SLINKY

MATERIALS

Slinky.
Meter stick.
Polaroid camera (optional).
Stopwatch.

PROCEDURE

1. Place slinky on lab table. Hold one end fixed while generating single transverse pulse at other end. Sketch shape of generated pulse and of reflected pulse.

2. Produce both transverse and longitudinal pulses in slinky and describe difference.

3. Generate transverse periodic wave in slinky, and determine its wavelength: place meter stick near slinky, and take a picture of wave with polaroid camera. Use stopwatch to determine period (T) of wave. Calculate velocity of wave, using Equation 2.

4. Adjust frequency of periodic waves until standing-wave pattern is observed. Draw sketch of standing wave pattern.

SUPERPOSITIONING OF SOUND WAVES

MATERIALS

Two identical loudspeakers.
Two sine-wave generators to drive the speakers.
Microphone.
Oscilloscope or portable tape recorder.
Laboratory stands for mounting speakers.
Connecting wires.

PROCEDURE

1. Operate two identical loudspeakers connected in same way to a sine-wave generator. A small amplifier may be necessary. Connect microphone to oscilloscope in order that sound generated by loudspeakers can be viewed on oscilloscope screen. Position loudspeakers facing each other (Figure 24). Adjust distance between speakers and frequency until a measurable standing-wave pattern is produced between them. Detect standing-wave pattern by observing changes in amplitude (nodes and antinodes) of wave on oscilloscope as microphone is moved from one speaker to the other on a line between them. Have your instructor verify your results.

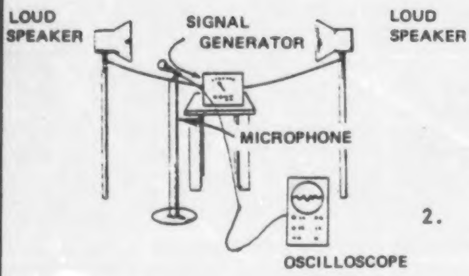


FIG. 24

2. Rotate loudspeakers 90° to face same direction. Move microphone throughout space in front of speakers, and identify areas of constructive interference and areas of destructive interference. Have your instructor verify your results.

3. Operate each speaker from a separate signal generator. Vary the frequency of one signal generator until you hear beats. The oscilloscope and microphone are recommended, but are unnecessary for this observation. When two signal generators are adjusted

in frequencies very close to each other, your ear should be able to detect beats. Is the beat frequency equal to the difference between the two oscillator frequencies? Have your instructor verify your results.

SELF CHECK

1. Draw and label transverse waves and longitudinal waves such as those you produced in a slinky in Demonstrations section. Explain major differences and similarities between these two types of waves.

2. Define these terms:

   Propagating medium.
   Wavelength.
   Frequency.
   Period.
   Displacement.
   Amplitude.
   Phase.
   Standing wave.
   Constructive interference.
   Destructive interference.
   Beats.

3. a. Calculate the frequency for a light wave whose wavelength is 555 nanometers. Use 3×10^8 m/s as the speed of light.
   b. What is the wavelength of air of a sound whose frequency is 980 Hz. Consider the speed of sound in air to be equal to 1130 ft/s.

4. Relate a sine wave to a circle by drawing a sine-wave sketch having a reference circle as given in the text. Explain, in your own words, how the $2\pi f$ in the sine wave formula is related to circular motion.

5. Draw an accurate sine-wave sketch of current leading voltage by 60°.

6. Explain in detail the meaning of the "superposition principle."

7. Describe and give examples of waves in the following energy systems:

   a. Mechanical.
   b. Fluidal.
   c. Electrical.

8. If the velocity of a wave along a string is 10 m/s and the wavelength is 5 cm, find the frequency of oscillation and the period associated with the wave?

9. At what frequency does a sinusoidally varying current alternate if at $t = 0$, the current is zero and it reaches its first maximum at $t = 1 \times 10^{-3}$ s?

10. What is the frequency of an alternating voltage if the instantaneous voltage is given by $V_t = V_{max} \sin (337\ t)$? (t is in seconds.)

11. Find the displacement of a wave at t = 6 sec, x = 10 m, if the wavelength is 3 cm and the frequency is 80 Hz. The amplitude of the wave is 0.5 cm.

12. At time t and distance x a wave has a displacement of 2 cm and a second wave has a displacement of -1 cm. What would be the displacement of a wave which is the superposition of the two waves at the same time t and distance x?

13. Two sound waves travel through air. If the frequency of one wave is 1000 Hz and the wavelength is 1 meter, find the frequency of a wave travelling through air under the same condition if its wavelength is 0.7 meters?

14. What is the maximum current flowing through an a.c. circuit if at time t = 17.37 seconds, the instantaneous value of the current is 20 mA and the frequency is 60 Hz?

15. Write the corresponding relation for current if the instantaneous voltage as a function of time is,

$$V_t = V_{max}\ (377\ t - \pi/2)$$

and the current leads the voltage by $\pi/4$ radians.

16. Two waves are identical except that the amplitude of one wave is twice that of the other. What is the ratio of the energies transferred by the two waves?

17. What direction is a wave propagating (± x direction?) if the equation for the displacement is

$$y = A \sin 2\pi\ (20\ x - 60\ t)?$$

(Hint: The direction of propagation of a wave is the direction that a point of constant phase (or displacement) moves. So as t increases, must x increase or decrease if the phase, $2\pi\ (20\ x - 60\ t)$ is to remain constant?)

18. What are the possible frequencies of an alternating current if at $t = 3.62 \times 10^{-2}$ seconds $\frac{I_t}{I_{max}} = 0.6$?

19. If the angular frequency of a wave is 370 rad/s, find the period of the wave.

20. Show that the equation

$$y = \sin \frac{2\pi}{\lambda} (x - vt)$$

is equivalent to Equation 4.

21. An organ pipe closed at both ends is 4 meters long. What is the longest possible wavelength of any standing wave which would fit into the pipe? (Hint: for a closed organ pipe a standing wave has a node at each end of the pipe.)

22. A simple pendulum swings through 40 complete oscillations in 6 seconds. What is the period and frequency of the motion of the pendulum?

23. Wave displacement as a function of time is often given as,

$$y = A \sin (Kx - wt).$$

Where K is a quantity called the wave number. Express K in terms of λ by comparing this equation with Equation 4.

24. A particle oscillates between $y = +2m$ and $y = -2m$. Its position as a function of time is given by $y = A \sin wt$ where A is the amplitude of the motion and w is the angular frequency. If 3 seconds elapse between successive times that the particle passes through $y = 0$, find w and A.

25. At what y value does the particle in Problem 24 have its maximum velocity.

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See especially experiments 29, 30, 31, and 50.

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TIME CONSTANTS

CHAPTER 12-0

In many practical situations, it is important to know how swiftly a system will give up or accept energy, will change its temperature, or will return to equilibrium after some initial displacement. For example, how much time is required for a heated piece of metal at some temperature T to cool to room temperature? How much time is required for a rotating record-player table to stop completely? How long must a battery be charged?

Some systems may not reach an equilibrium, or final state, until an extraordinary length of time has elapsed. As an example, a capacitor never becomes fully charged; however, in practice it may come so close to being charged in a given time that it can be considered "fully" charged.

Time constants — intervals of time during which a system undergoes a given change — are used to describe how fast systems are changing. If the time constant for a changing system is known, one can predict time required for the complete change to take place; for example, time required for a capacitor to reach 99% of a fully-charged condition. In this module, time constants will be shown to be common to different energy systems and to serve, therefore, as a unifying concept.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following terms:

  Steady state.
  Transient.
  Damping.

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Time constant.
Half-life.
Decay constant.

- Draw and label a graph showing an exponentially-decaying function. Include on the graph the time constants $T_{1/2}$ and $T_{1/e} \equiv \tau$. Also, write an equation for the function shown on the graph.

- Given the initial temperature of a hot body, the ambient temperature of its surroundings, and the thermal time constant of the system, determine the temperature of the body after a specified time interval.

- Given the number of radioactive atoms in a sample and the decay constant, determine the number of atoms remaining after a specified time interval.

- Given the values of resistance, capacitance, and applied voltage in an RC electrical circuit, determine the time constant for the circuit, the time required for the capacitor voltage to reach 99% of the applied voltage, and the circuit current and capacitor voltage after a specified time interval.

- Explain how the concept of time constants can be applied to the following energy systems; give a specific example in each case:

  Mechanical translational.
  Mechanical rotational.
  Fluidal.
  Electrical.
  Thermal.
  Optical.
  Nuclear.

DISCUSSION

Concept Module 3-0 discussed rates, all of which were constant and, therefore, did not vary with time. For example, if one gallon per minute was the given flow rate of a fluid through a pipe at any given time, flow rate at any other time also would be one gallon per minute. Thus, flow rate did not change with time. Processes that do not change with time are called "steady state."

While a process is changing with time, but is approaching a steady state, the process is called "transient." Although a process may be changing, it often does so in a smooth and continuous fashion. Consider the example of a thermometer heated to a temperature of 100°C. If temperature of liquid in the bulb of the thermometer is plotted as a function of time, a curve is obtained (Figure 1) as it cools to room temperature.

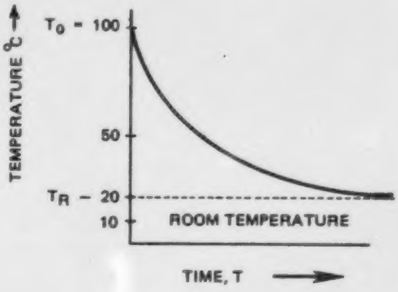


FIG. 1

When the thermometer begins to cool, the temperature difference between the liquid in the bulb and the room air is 80C°. Since heat flow rate is proportional to temperature difference, this establishes an initial heat flow rate for the system. As the thermometer cools, the temperature difference (forcelike quantity) producing the heat flow also decreases. Thus, when the thermometer bulb temperature has dropped to 60°C, the temperature difference is only 40C°; and the heat flow rate from the bulb to the air is one-half its initial value, resulting in slower cooling of the bulb. This process continues, with an ever-decreasing rate of temperature change, until the thermometer bulb reaches room temperature after some lengthy time.

The temperature of the liquid in the thermometer is given at any time t by Equation 1.

$$T = T_{amb} + [T_o - T_{amb}]\, e^{-kt} \qquad (1)$$

where

T = temperature of thermometer at time t.
T_o = temperature of thermometer at time t = 0.
T_{amb} = ambient temperature (room temperature T_R in this case).
k = decay constant with dimensions of inverse time.
e = exponential base = 2.71825.
t = time.

When $t = 0$, Equation 1 reduces to —

$$T = T_R + (T_o - T_R)\ e^{-o} = T_R + (T_o - T_R)(1),$$

since e^o equals unity. Finally then, we get $T = T_o$, which confirms the claim above that T_o is the temperature at time $t = 0$. As time goes on, liquid in the thermometer approaches room temperature in a smooth and continuous fashion, as shown in Figure 1. Curves that decrease in accordance with Equation 1 are called "exponential."

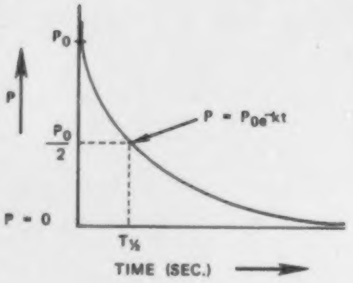


FIG. 2

The curve shown in Figure 2 indicates that some parameter P is an exponentially-decreasing function of time in accordance with the equation $P = P_oe^{-kt}$. Notice that the curve is similar to the temperature profile plotted in Figure 1.

The curve in Figure 2 begins with a peak value of $P = P_o$ at time $t = 0$. The curve approaches $P = 0$ exponentially as time increases indefinitely. Notice that P does not reach $P = 0$ until time $t = \infty$! But, for all practical purposes, it almost reaches $P = 0$ long before. Since a finite time when it reaches zero cannot be given, a convenient time interval called $T_{1/2}$, which is the time for the system to decrease from $P = P_o$ to $P = P_o/2$, is introduced. Because the initial value P_o has been reduced by a factor of one-half, the time interval $T_{1/2}$ is called the "half-life" for the changing parameter P. Observe carefully the values of $P_o/2$ and corresponding point $T_{1/2}$ in Figure 2. Half-life $T_{1/2}$ is a convenient time interval — referred to as a "time constant" — for gaging rate of change in systems that follow an equation such as $P = P_oe^{-kt}$. It often is used in nuclear systems in which radioactive decay, an exponentially-decaying process following an equation like $P = P_oe^{-kt}$, is studied.

The time required for a system to change from any P to P/2 is indicated by Equation 2.

$$T_{1/2} = \frac{0.693}{k} \qquad (2)$$

where

k = constant in equation of the type $P = P_oe^{-kt}$.
$T_{1/2}$ = time required for system to change from any P to P/2.

Another commonly-used time constant is defined as the time interval during which the initial parameter P_o decreases to P_o/e (rather than to $P_o/2$), where e is the naperian base = 2.71828.

This time constant, denoted $T_{1/e} \equiv \tau$, is equal to $1/k$, where k is the same constant defined earlier.

$$T_{1/e} \equiv \tau = \frac{1}{k} \qquad (3)$$

The value for $\tau = 1/k$ easily is obtained as follows:

$$P = P_o e^{-kt}$$

(1) Set $P = \frac{P_o}{e}$ at $t = \tau$: $\frac{\cancel{P}_o}{e} = \cancel{P}_o e^{-k\tau}$

(2) So that — $\frac{1}{e} = e^{-1} = e^{-k\tau}$

(3) This arrangement requires — $-1 = -k\tau$ or $\tau = \frac{1}{k}$.

Since $\tau = 1/k$ is the time for $P = P_o$ to decrease to $P = P_o/e$, and $P_o/e = P_o/2.718 \simeq 0.37\ P_o$, τ is the time for P_o to decrease to $0.37\ P_o$. The time constant $T_{1/2}$, discussed previously, reduces P_o to $0.50\ P_o$.

Times at which the parameter P_o reached $1/2\ P_o$ or $0.37\ P_o$ are shown in Figure 3. Either time constant could be used to characterize the rate at which the parameter is decreasing.

Example A illustrates use of $T_{1/2}$ and τ in a system undergoing an exponential decrease.

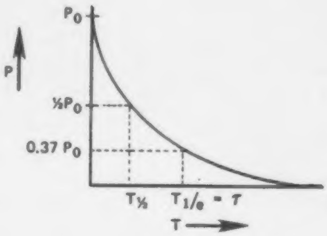


FIG. 3

EXAMPLE A: HOT BODY COOLING TOWARD ROOM TEMPERATURE

Given: A hot block of iron cools according to the equation $T = T_R + (T_o - T_R)\ e^{-kt}$, where $T_R = 20°C$, $T_o = 700°C$ and $k = 0.04/\text{min}$.

Find:
- (a) What time constant $T_{1/2}$ equals and temperature at that time.
- (b) What time constant $T_{1/e} = \tau$ equals and temperature at that time.
- (c) Final temperature after a time interval equal to $5\ \tau$ (that is, $t = 5\ \tau$) has elapsed.

Solution: (a) $T_{1/2} = \frac{0.693}{k} = \frac{0.693}{0.04/\text{min}} = 17.3 \text{ min}$

After 17.3 min, temperature $T = \frac{T_o}{2} = \frac{700}{2} = 350°\text{C}$

(b) $\tau = \frac{1}{k} = \frac{1}{0.04/\text{min}} = 25 \text{ min}$

After 25 min, temperature $T = \frac{T_o}{e} = 257.5°\text{C}$

(c) $T = T_R + (T_o - T_R)\ e^{-kt}$

$t = 5\ \tau = 5\ (25) = 125 \text{ min}$

$T = 20 + (700 - 20)\ e^{-(0.04)(125)}$

$= 20 + 680\ e^{-5}$

$= 20 + 680 \cdot (0.0067) = 20 + 4.6$

$T = 24.6°\text{C}$

Thus, after five time constants or two hours and five minutes, temperature of the hot body has dropped from 700°C to 24.6°C. It still has $\simeq$ 5°C to go to reach 20°C. This change will require a long time, since the decrease is now very slow. However, note that, in five time constants, temperature already has dropped 675°C of the 680°C it has to go to reach room temperature at 20°C. This is more than 99% of the total change.

In a similar manner, an exponentially-increasing function is shown in Figure 4. The changing parameter q begins at zero time t = zero and rises toward Q as time increases; again the time constant $\tau = 1/k$ is useful in describing the process. Since $q = Q\ (1 - e^{-t/\tau})$, $q = Q\ (1 - e^{-1})$ when $t = \tau$. Thus, the parameter q has reached $q = Q\ (1 - 0.37) = 0.63\ Q$, or 63% of its final value Q after one time constant.

The parameter q never reaches its maximum value Q, just as the decreasing P parameter never reaches zero. One can show that both exponentially-increasing and decreasing functions reach 99% of their final values after 5 time constants, $t = 5\ \tau$, as was shown in Example A.

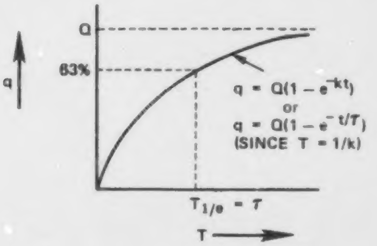


FIG. 4

RADIOACTIVE DECAY

Radioactive decay is spontaneous disintegration or transformation of one nuclide into another. Radioactive decay results in a decrease, with time, in the number of original radioactive atoms in the sample, because those radioactive nuclides that undergo decay usually are changed into more stable types of nuclides. To determine exactly when one particular radioactive atom will decay is impossible, because the decay process is random. However, because the total number of original radioactive atoms present in any sample is extremely large, this number undergoes a predictable, continuous change. The number of atoms undergoing decay in any specified time period, and the amount of radioactivity produced, can be calculated with reasonable accuracy.

An important property of a radioactive nuclide is its half-life. Half-life is the time in which half the atoms of a particular radioactive nuclide decay to another nuclide. Each radioactive nuclide has its own characteristic half-life.

Assume that one has a radioactive source and that some means, such as a Geiger counter, of counting the number of atoms that disintegrate in a given time interval is available. If one plots the number of original atoms present at a given time versus time on a linear graph (Figure 5), the curve obtained indicates an exponential, or logarithmic, relationship. If the number present versus time is plotted on semilog paper (Figure 6), a straight line is obtained, which, again, indicates that radioactive decay is an exponential process. When there is a constant fractional decrease in the parameter during equal intervals of time, decay is exponential. Although the same number of atoms do not disintegrate during each unit of time, the same fraction of the atoms present decay during each unit of time. This behavior leads to the general exponential relationship for radioactive decay illustrated in Equation 4 and Example A.

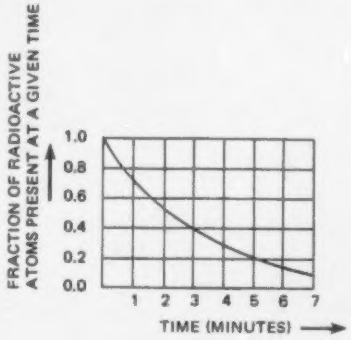


FIG. 5

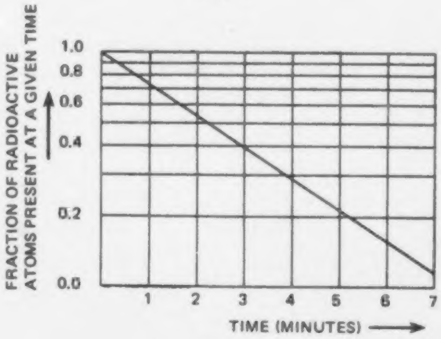


FIG. 6

$$N = N_o e^{-\lambda t} \qquad (4)$$

where

N = number of undecayed atoms left at time t.
N_o = number of atoms originally present at $t = 0$.
λ = decay constant, s^{-1}, min^{-1}, $hour^{-1}$, etc.

Example B illustrates use of Equation 4 in a realistic radioactive-decay problem.

EXAMPLE B: RADIOACTIVE DECAY RATE

Given: A radioactive sample contains 10^{22} atoms; the decay constant is 0.1 hr^{-1}.

Find:

(a) Number of original radioactive atoms remaining 4.0 hours from now.

(b) Time constant τ.

Solution: (a)

$$\lambda = 0.1 \text{ hr}^{-1}$$

$$N_o = 10^{22} \text{ atoms}$$

$$t = 4 \text{ hours}$$

$$N = N_o e^{-\lambda t}$$

$$= (10^{22} \text{ atoms}) e^{-0.1 \text{ hr}^{-1} \times 4.0 \text{ hr}}$$

$$= 10^{22} \text{ atoms } e^{-0.4}$$

$$= 0.67 \times 10^{22} \text{ atoms}$$

$$N = 6.7 \times 10^{21} \text{ atoms}$$

(b) Time constant $\tau = \frac{1}{k} = \frac{1}{\lambda}$

Therefore — $\tau = \frac{1}{0.1/\text{hr}} = 10 \text{ hours}$

After 50 hours (five time constants), 99% of the atoms will have decayed, and only 1% will remain.

The activity of a given sample usually is more important than the number of atoms present. Activity of a sample is defined by Equation 5 and illustrated by Example C.

$$A = \lambda N \tag{5}$$

where

A = activity, counts per minute (c/m) or disintegrations per second (d/s).

λ = decay constant, s^{-1}, min^{-1}, $hour^{-1}$, etc.

N = number of undecayed atoms.

EXAMPLE C: DISINTEGRATION OF A RADIOACTIVE MATERIAL

Given: At a given instant of time t, a radioactive sample contains 10^{18} atoms and has a decay constant of $10^{-7}\ s^{-1}$.

Find: Activity in disintegrations per second at that time.

Solution: $N = 10^{18}$ atoms at time t

$\lambda = 10^{-7}\ s^{-1}$

$A = \lambda N$

$= 10^{-7}\ s^{-1}\ 10^{18}$ atoms

$= 10^{11}$ atoms/s

$A = 10^{11}$ disintegrations per second

Notice that activity at any time depends on number of undecayed atoms remaining at that time, and so must decrease with time.

Decay constant λ is closely related to concept of half-life of a radioisotope. Half-life, $T_{1/2}$, is defined as time required for a given isotope to decay to one-half its initial quantity or activity. Consider an initial activity of A_o. At some time $t = T_{1/2}$, activity will be equal to $A_o/2$. Relationship for activity or disintegration rate A as a function of time is similar to the relation for number of atoms N as a function of time, given by $N = N_o e^{-\lambda t}$ (Equation 4). Thus, activity A has this form:

$$A = A_o e^{-\lambda t}$$

After one half-life, $t = T_{1/2}$, $A = A_o/2$ so that the following is derived:

$$\frac{1}{2} A_o = A_o e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

Take the natural log of both sides:

$$\ln\left(\frac{1}{2}\right) = -\lambda T_{1/2}$$

$$\lambda T_{1/2} = 0.693$$

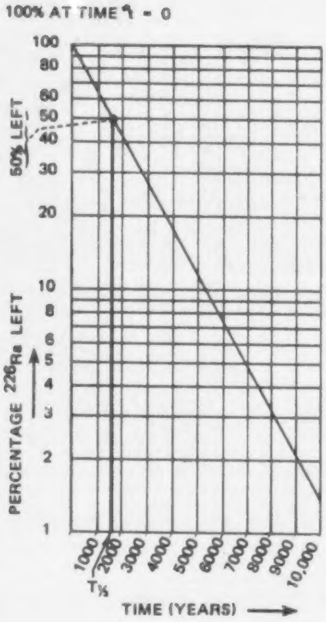


FIG. 7

Rearrange and solve for $T_{1/2}$:

$$T_{1/2} = \frac{0.693}{\lambda}$$

This relationship is similar to development for $\tau = 1/k$ (Equation 2). Measured half-lives range from about 10^{-14} years (3×10^{-7} s) for ^{212}Po to about 10^7 years for ^{209}Bi. A decay curve for radium-226 indicating a half-life of 1600 years is illustrated in Figure 7.

MECHANICAL SYSTEMS

TRANSLATIONAL

If a mass is attached to a spring and started in vertical motion (left portion of Figure 8), displacement of the mass as a function of time will result in a graph like that shown in Figure 8.

The affect of friction usually is neglected in a first discussion of this motion. Clearly, if no friction is present, vertical vibration will continue indefinitely. Actually, a weight on a spring will not oscillate indefinitely; since friction always is present, amplitude will decrease regularly, and the mass eventually will come to rest. The motion is said to be damped out by friction and is called "damped motion." How fast the weight comes to rest depends upon how much it is damped.

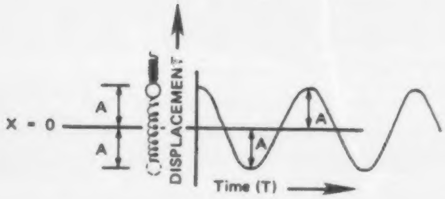


FIG. 8

If some small amount of friction, such as air drag, is present, the displacement-time graph shown in Figure 8 resembles that shown in Figure 9.

The time constant $T_{1/2}$ of this system is defined as time required for the amplitude of the oscillations to decrease from x_0 to $x_0/2$. The time constant $T_{1/e} = \tau$ is defined as time required for the amplitude of the oscillations to decrease to x_0/e or $0.37\ x_0$. $T_{1/2}$ and τ are illustrated in Figure 9.

ROTATIONAL

The balance wheel of a watch represents a mechanical system in which oscillation is damped due to frictional torques. The balance wheel of a watch that was originally at position $\Theta = 0°$ at time $t = 0$ is rotated to $\Theta = 0.1$ rad (57.3°) and released. The wheel then oscillates back and forth around the position $\Theta = 0°$. If angular displacement of the wheel is plotted as a function of time, the displacement-time curve appears as shown in Figure 10. (Notice carefully that for this motion, displacement is measured in radians or degrees.) The time constant $T_{1/2}$ of this balance wheel is defined as time required for the initial angular displacement $\Theta = 0.1$ rad to damp out (due to frictional torques) to 1/2 this initial value, or 0.05 rad. The time constant τ for this motion is time for the initial amplitude of 0.1 rad to damp out to a value equal to 0.37 (0.1 rad) or 0.037 rad. Both of these values are shown in Figure 10.

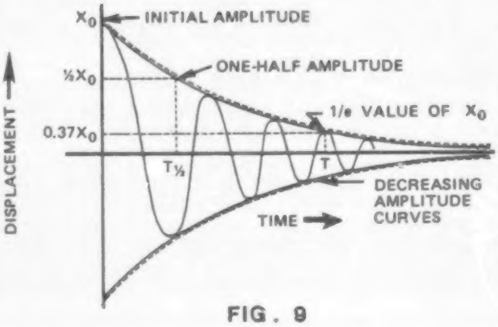


FIG. 9

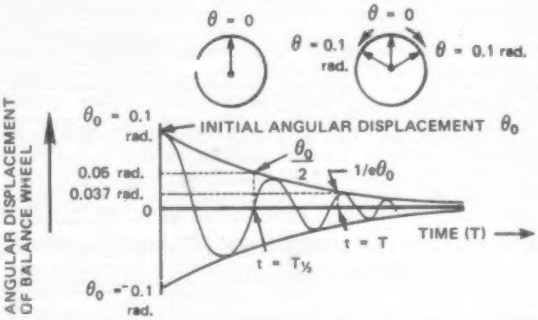
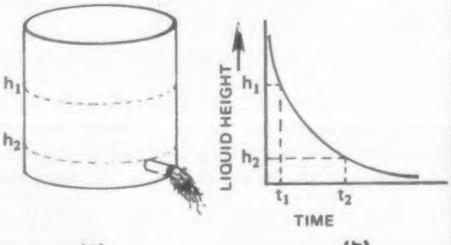


FIG. 10

FLUIDAL SYSTEMS

Time constants are not often applied in fluidal systems, but are very useful in one specific problem. This problem concerns the time required to empty a tank because of the influence of gravity. Figure 11 illustrates such a problem. The flow rate from the tank depends upon the pressure at the bottom of the tank, which, is turn, depends upon the height of the liquid above the opening. At t_1 of Figure 11a, the height h_1 produces a certain flow rate. At t_2 the height has decreased to h_2, and the flow rate from the tank has decreased proportionally. Thus, the liquid level drops as an exponential curve, as shown in Figure 11b.



(a) (b)

FIG. 11

ELECTRICAL SYSTEMS

Now let us consider the process of charging and discharging a capacitor. We shall consider the two special cases illustrated in Figure 12.

In Figure 12a, the switch is ready to be closed with the capacitor completely uncharged. In Figure 12b, the capacitor already is charged to a voltage V_s, and the closing of the switch short circuits the capacitor; thus, 12a illustrates a charging circuit and 12b is a discharging circuit. R represents total resistance of the circuit.

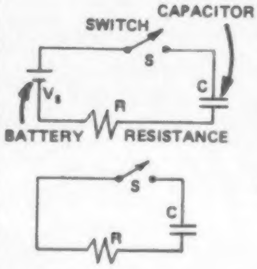


FIG . 12

When the switch is closed in Figure 12a, voltage across the capacitor as a function of time is given by Equation 6.

$$V_c = V_s \left(1 - e^{-t/RC}\right) \qquad (6)$$

where

V_c = voltage across capacitor.

V_s = voltage to which capacitor is charged.

Charging current as a function of time is given by Equation 7.

$$I = \frac{V_s}{R} e^{-t/RC} \qquad (7)$$

where

I = charging current.

R = resistance.

Charging current and voltages are shown in Figure 13. Comparing Equation 7 with the general form $P = P_o e^{-kt}$, we see that $k = 1/RC$. Then, since $\tau = 1/k$ in general, τ must equal RC for Equation 7. Thus, setting $RC = \tau$ in Equations 6 and 7 yields Equation 8.

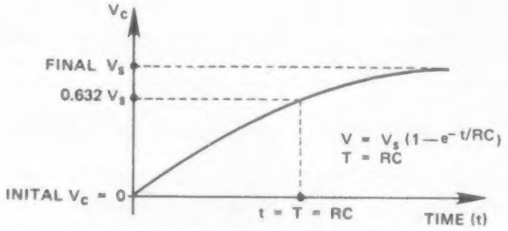
$$V_c = V_s \left(1 - e^{-t/\tau}\right) \qquad (8)$$

where

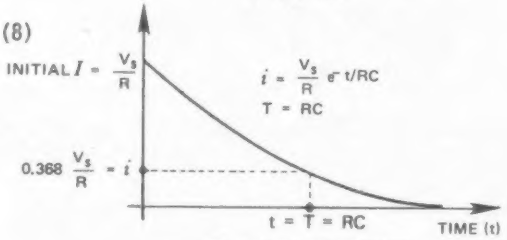
τ = RC = time constant.

At a time equal to one time constant, Equation 8 becomes —

$$V_c = V_s \left(1 - e^{-1}\right), \text{ and } I = \frac{V_s}{R} e^{-1}$$



(a) CHARGING VOLTAGE VERSUS TIME



(b) CHARGING CURRENT VERSUS TIME

FIG . 13

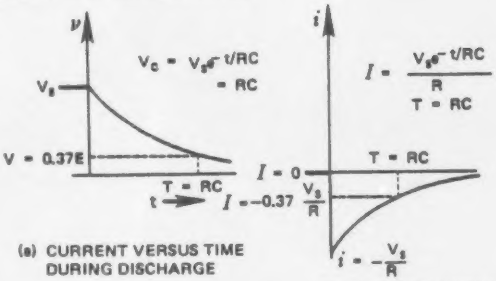
$$V_c = V_s\ (1 - 0.368),\ \text{and}\ I = \frac{V_s}{R}\ (0.368)$$

or

$$V_c = 0.632\ V_s,\ \text{and}\ I = 0.368\ \frac{V_s}{R}$$

In other words, at the end of one time constant $\tau = RC$, voltage across the capacitor is 63% of supply voltage, and current has dropped to 36.8% of its original value.

During the discharge phase shown in Figure 12b, voltage across the capacitor and the circuit current are given by the graphs shown in Figures 14a and 14b, respectively. As before, the time constant is $\tau = RC$. As indicated in Figure 14, after a time equal to one time constant $\tau = RC$, voltage decreases from V_s to 37% of V_s, while current decreases from a maximum magnitude of V_s/R to 37% of V_s/R. (The negative sign indicates that discharge current flows in opposite direction to the direction previously assigned to charging current.) Example D illustrates how the time constant is used in practical problems.



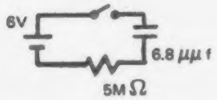
(a) CURRENT VERSUS TIME DURING DISCHARGE

(b) CURRENT VERSUS TIME DURING DISCHARGE

FIG. 14

EXAMPLE D: TIME CONSTANT CALCULATION IN AN RC CIRCUIT

Given: From Figure 12a, resistance $R = 5\ \text{m}\Omega$, and capacitance $C = 6.8\ \mu\mu\text{f}$, and $E = 6$ V.



Find: Time constant of this circuit. What does this time constant indicate?

Solution: $T = RC = 5 \times 10^6 \times 6.8 \times 10^{-12} = 34.0 \times 10^{-6}$ s, or

$T = 34\ \mu$s.

This value means that voltage across the capacitor reaches 63.2% of 6 V or $v_c = 3.792$ V after 34 µs. It also means that current at the same time has a value of —

$$I = \frac{V_s}{R}\ (0.368) = \frac{6\ \text{V}}{5 \times 10^6}\ 0.368 = 0.44 \times 10^{-6}\ \text{amps, or}$$

$I = 0.44\ \mu$amps

THERMAL SYSTEMS

Hot objects cool down to the temperature of their surroundings. If temperature difference ΔT between an object and its surroundings is not too great, the rate of cooling is approximately proportional to temperature difference between the object and its surroundings. This rate results in the relationship given by Equation 9.

$$\Delta T = \Delta T_o e^{-kt} \tag{9}$$

where

ΔT = temperature difference between object and its surroundings at time t.

ΔT_o = temperature difference between object and its surroundings at time $t = 0$.

k = constant.

t = time.

(Note: At this point, this equation is identical to Equation 1.)

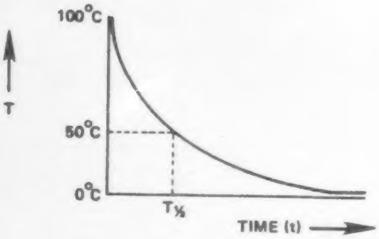


FIG. 15

A mercury-in-glass thermometer is placed in boiling water for a few minutes and then removed. Assume that temperature of the surroundings is at the freezing point, 0°C. Temperature indicated on the cooling thermometer is plotted as a function of time in Figure 15.

Since surrounding temperature is taken as 0°C, the time constant $T_{1/2}$ of this system is defined as time required for the thermometer to reach 50°C or one-half the boiling temperature of water, 100°C. This is shown in Figure 15. The student similarly should indicate the time constant τ and its meaning on Figure 15.

OPTICAL SYSTEMS

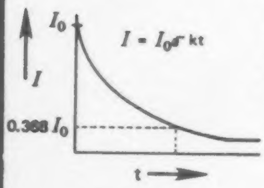


FIG. 16

Phosphorus used to coat a TV screen gives off light that decays at a certain rate. When the TV set is turned off, the screen continues to glow for several minutes. If intensity of light from a fluorescent or phosphorescent screen were measured and plotted as a function of time, a curve similar to that in Figure 16 would be obtained.

Equation 10 is for light intensity.

$$I = I_o e^{-kt} \tag{10}$$

where

I = intensity at time t.
I_o = intensity at time $t = 0$.
k = decay constant.

Since this is a standard exponentially-decreasing function, the time constant in this case must be the following:

$$\tau = T_{1/e} = \frac{1}{k}$$

At this time, light intensity has dropped to 36.8% of its original brightness. After five time constants ($5\ \tau$), less than 1% of the original intensity remains; therefore, the screen is almost dark. Characteristic time constants for phosphorescent decay are in the order of milliseconds to microseconds.

SUMMARY

Exponential change of a quantity, whether increasing or decreasing, is described conveniently by use of a time constant for the process. Two such time constants are in common use, and both have been described in this module. In either case, a small time constant for the system characterizes a relatively rapid change, and a large time constant characterizes a relatively slow change.

The time constant provides another quantity that finds application in all basic physical systems. In any system, when a quantity is changing and when its rate of change is proportional to its present value, an exponential change occurs. In this module, exponential changes and corresponding time constants have been shown to apply to the following systems:

- Mechanical-translational system, in which rate of change of amplitude of linear oscillations is proportional to amplitude of any time.
- Mechanical-rotational system, in which rate of change of angular displacement of oscillations is proportional to angular displacement at any time.
- Electrical system, in which rate of change of voltage (or current) on a charged capacitor is proportional to voltage (or current) at any time.

- Thermal system, in which rate of change of temperature difference of a body with its ambient temperature is proportional to temperature difference at any time.
- Optical system, in which rate of change of light intensity is proportional to time.
- Nuclear system, in which rate of disintegrations of radioactive atoms is proportional to number of radioactive atoms present at any time.

These cases are a small sample of similar situations in physics. An understanding of exponential change in terms of a time constant helps the technician to unify all such cases by abstracting from such diverse physical processes their essential similarity in behavior.

MECHANICAL SYSTEM - LINEAR TRANSLATIONAL

MATERIALS

Stopwatch.
Spring with weight (spring should be quite flexible to give a large amplitude).

PROCEDURE

1. Suspend a weight from a flexible coiled spring, as shown in Figure 17.
2. Indicate, on a meter stick, a positive 10 cm above and 10 cm below equilibrium position by symbols $+x_o$ and $-x_o$. This distance does not have to be 10 cm; but the larger the displacement value, the easier the measurements will be. Also label positions $+0.5\ x_o$ and $-0.5\ x_o$.
3. Move weight up to position $+x_o$, and release weight. Be careful not to give weight any side-to-side motion.
4. Start stopwatch at instant the weight is released, and carefully note times when oscillations reach $\pm0.9\ x_o$, $\pm0.8\ x_o$, $\pm0.7\ x_o$, $\pm0.6\ x_o$, $\pm0.5\ x_o$, $\pm0.4\ x_o$, $\pm0.3\ x_o$, $\pm0.2\ x_o$, $\pm0.1\ x_o$, etc.
5. Plot data as shown in Figure 18.
6. On Figure 18, indicate time when value of oscillation reached 1/2 its original value and label this time $T_{1/2}$. What is experimental value of $T_{1/2}$?
7. In a time equal to 5 $T_{1/2}$, amplitude should have decreased to about $0.03\ x_o$. Calculate 5 $T_{1/2}$, and check whether or not it is greater than time required to decrease amplitude to $0.1\ x_o$. (It should be!)

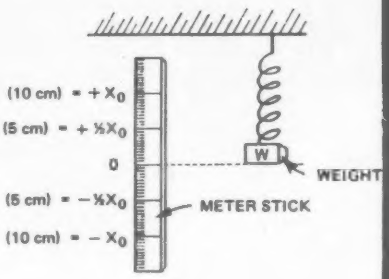


FIG. 17

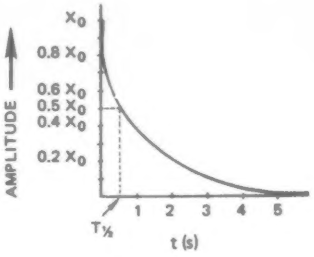


FIG. 18

MECHANICAL SYSTEM - ROTATIONAL

MATERIALS

Torsion pendulum flywheel on shaft.

PROCEDURE

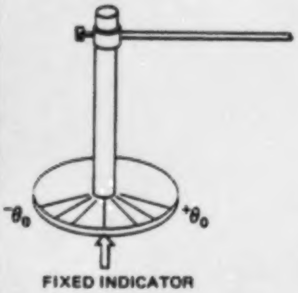


FIG. 19

1. Suspend a circular weight on a stiff shaft, as shown in Figure 19.
2. Rotate disc to a predetermined angle $\pm\theta_0$, and carefully release disc.
3. Record the time required for amplitudes of oscillation to reach several intermediate angles before coming to rest. Record time here: __________.
4. Plot this data as in Figure 20.
5. Draw a horizontal line from 0.5 θ_0 until it intersects the curve. Draw a vertical line from this point to time axis. Read time constant $T_{1/2}$ off graph.
6. When will the oscillative disc have decreased its amplitude of oscillation to a value of 3% of its original amplitude?

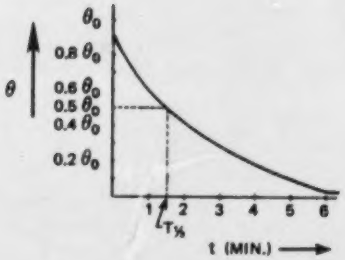


FIG. 20

THERMAL SYSTEM

MATERIALS

Thermometer.
Beaker.
Bunsen burner.

PROCEDURE

1. Bring a container of water to boil.
2. Note room temperature.
3. Place a thermometer in water, and let it come to equilibrium.
4. Remove thermometer, and immediately begin reading temperature versus time at 5°C intervals.
5. In Data Table, record temperature versus time.
6. Plot data obtained as in Figure 21.
7. Determine time constant $T_{1/2}$ and $\tau = T_{1/e}$ for this thermometer.

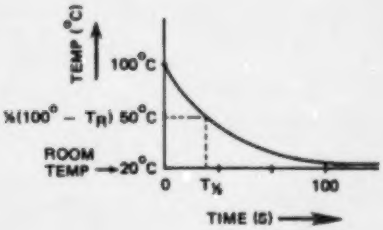


FIG . 21

DATA TABLE			
Temperature °C	Time (s)	Temperature °C	Time (s)
100	0	60	
95		55	
90		50	
85		45	
80		40	
75		35	
70		30	
65		25	
		T_R = 20	

MATERIALS

6-volt battery.
Stopwatch.
Resistors (15 kΩ and 25 kΩ).
Capacitor (2000 µf - electrolytic).
Switch (DPDT).
Microammeter (full-scale; at least 400 microamps).

PROCEDURE

1. Connect circuit elements as shown in Figure 22 to terminals of a double-pole, double-throw (DPDT) switch. CAUTION: Since capacitor is electrolytic, polarity must be observed. Have instructor check wiring.

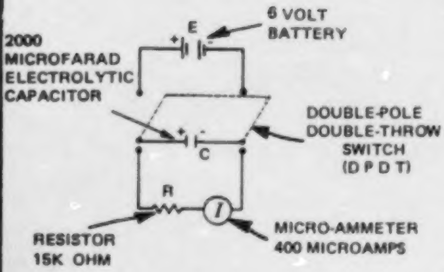

6 VOLT BATTERY
E
+
2000 MICROFARAD ELECTROLYTIC CAPACITOR
DOUBLE-POLE DOUBLE-THROW SWITCH (D P D T)
+
-
C
R
I
RESISTOR 15K OHM
MICRO-AMMETER 400 MICROAMPS

FIG. 22

2. Use 15-kΩ resistor provided. Charge capacitor for a few seconds by throwing switch toward battery-connected terminals. Discharge capacitor by reversing throw of switch, at same time starting time measurement with stopwatch. Take data on current read by meter versus time elapsed. If current decreases too rapidly to make all measurements during one discharge, process can be repeated, each time measuring one or more current-versus-time data points.

3. Repeat for a 25-kΩ resistor.

4. Plot current versus time and, from graph for each resistor, determine time constant τ. How does experimental value compare with theoretical?

OPTICAL SYSTEM

MATERIALS

Hook-up wire.
TV or oscilloscope.
Optical power meter.
Strip chart recorder.

PROCEDURE

1. Turn on TV set or oscilloscope, and allow sufficient time for warm-up (showing a bright picture tube).
2. Place optical power meter in front of screen, as shown in Figure 23.
3. Record output of optical power meter as a function of time on a chart recorder (Figure 24). Be sure to leave power meter at same position near TV set while taking readings. Do not change ambient lighting conditions while readings are being taken.
4. Determine time required for intensity of fluorescence to reach 50% of its peak value after turning it off.
5. Determine time constant $T_{1/2}$.

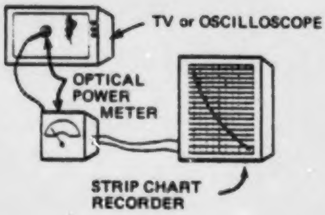


FIG. 23

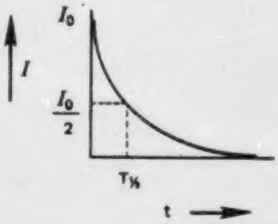


FIG. 24

SELF CHECK

1. Define the following terms:

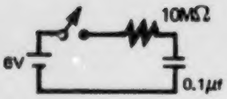
   Steady state.
   Transient.
   Damping.
   Time constant.
   Half-life.
   Decay constant.

2. Draw a graph for an exponentially decaying quantity P. Show on the graph the time constants $T_{1/2}$ and τ and the values of P at these times. Write an equation for the graph.
3. A hot body at 500°C is placed in a cooler at 0°C. After five minutes, the temperature is 400°C. Determine the following:
   - a. Decay constant k.
   - b. Time constant τ.
   - c. Temperature after two time constants ($t = 2\tau$).
   - d. Temperature after five time constants ($t = 5\tau$).

4. The common isotope of uranium ^{238}U has a half-life of 4.50×10^9 years. Determine the following:

   a. Decay constant λ.
   b. The fraction of original atoms remaining after 9.0×10^9 years.
   c. The number atoms of ^{238}U needed for an activity of 1 curie. (A curie is defined as an activity of 3.70×10^{10} d/s.)

5. Given the circuit shown in the adjoining sketch, determine the following:



   a. Time constant.
   b. The current through the resistor when the switch is closed.
   c. The current through the resistor after two time constants.
   d. The voltage on the capacitor after three time constants.
   e. The voltage on the capacitor after five time constants.

6. Explain the use of time constants in each of the following energy systems, and give an example in each system:

   Mechanical translational.
   Mechanical rotational.
   Fluidal.
   Electrical.
   Thermal.
   Optical.
   Nuclear.

7. An exponentially decreasing process has a half life of 0.231 seconds. What is the value of the time constant (τ) for this process?

8. What is the value of the current which is charging a capacitor if $V_s = 8$ V, R = 10 kΩ, and the capacitor has a value of 12 μf, 0.5 seconds after the charging process started?

9. A capacitor is being charged by a 12 V battery. What is the voltage across the capacitor at the instant the charging process begins. What is V_c an essentially infinite time later?

10. How much time would it take for the voltage across the capacitor in problem 8 to reach 4 volts? (Use the same circuit parameters as in problem 8.)

11. A variable resistor is used as the resistive element in a circuit designed to charge a capacitor. The resistor has a maximum value of 10,000 Ω and a minimum value of 10 Ω. What is the ratio of maximum to minimum time constant for this system?

12. What is the value of the constant K (see Equation 9) if a thermometer reads 30°C, 10 seconds after being removed from a pan of boiling water? Assume that the temperature of the surroundings is 20°C.

13. What is the half life for the thermometer of problem 12? What is the temperature reading of the thermometer two half lives after the thermometer is removed from the boiling water?

14. It requires 10 seconds for the current charging a 10 µf capacitor to decrease to $I_{max}(1/e)$. If I_{max} is 100 µA, what is the resistance in the circuit and the voltage of the source?

15. A 50 µf capacitor is to be charged by a 50 volt battery. After 10 seconds the voltage across the capacitor is 10 volts. What is the resistance in the circuit?

16. How long would it take for the voltage across the capacitor in problem 15 to reach 10 volts if the resistance in the circuit is doubled?

17. A tank of water is to be drained by opening a spigot located at the bottom of the tank. If it takes 20 minutes for the height of the water in the tank to drop to 4/5 of its original value, how much time will elapse (after opening the spigot) before the tank is half empty?

18. How much time will elapse before the tank of water in problem 17 becomes 99% empty? (K = 0.011/min)

19. What is the decay constant of a radioactive sample if the activity is 200 counts/sec when the total number of undecayed atoms is 2×10^8?

20. The activity of a radioactive sample is 25 (c/m) when there are 3×10^{15} undecayed atoms present. How many undecayed atoms are present 2×10^9 years later?

21. What is the decay constant for a radioactive sample if the half life is 12 years?

22. How long will it take for the tank of water in problem 17 to become completely empty?

23. What percentage of the original number of undecayed atoms remain undecyaed after 10 half lives?

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RADIATION

CHAPTER 13-0

Radiation is the process by which energy moves away from a source in a straight line. Sound waves, light waves, and alpha and beta rays are examples of radiation. Although these examples represent three distinct classes of radiation, each example moves energy radially away from some source, and all have common characteristics.

Sound waves are pressure variations and require a material medium for transmission. They are discussed in Concept Module 11-0, "Vibrations and Waves," and will be mentioned here only briefly. This module addresses mainly the topic of electromagnetic (EM) radiation — sometimes called "radiant energy." In this type of radiation, energy is transmitted by variations in electric and magnetic fields. Radiant energy includes gamma rays, x-rays, ultraviolet, visible, infrared, microwaves, radar, television, and radio waves. It is the basis of modern communication, of solar energy, and of sight itself. Since EM radiation is so important in our lives, most of this module will deal with this type of radiation. Alpha and beta (nuclear) particles also will be discussed briefly as examples of particle radiation in which energy is transmitted as kinetic energy of particles.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Describe in one or two sentences the basic properties of each of the following types of radiation:

  Sound.
  Light.
  Alpha and beta particles.

- Define electromagnetic radiation (radiant energy), and describe a simple experiment that illustrates how electromagnetic radiation can be created.
- List the frequencies in the electromagnetic spectrum from long wavelength EM waves of a.c. power (60 hertz) to gamma rays (10^{22} hertz), including each major part — radio, FM, television, radar, microwave, infrared, visible, ultraviolet, x-ray, and gamma ray.
- Given the equation $v = \lambda f$ — relating wave speed, wavelength, and frequency — determine the radiation frequency for any part of the electromagnetic spectrum.
- Given the equation $E = hf$ or $E = h\ c/\lambda$, determine the energy of different waves in the EM spectrum.
- Describe qualitatively the nature of an electromagnetic wave in terms of electric and magnetic fields; state what is always required to generate an EM wave; and explain how EM waves are propagated through empty space without benefit of an elastic medium.
- Describe a photon, and explain why both wave and particle-like (photon) phenomena are required to describe interaction of EM radiation with matter. Give examples in which the wave character is most useful in describing EM radiation and in which the photon character is most useful.
- Explain what is meant by the inverse-square law and how this law is used to describe the fall-off of EM radiation propagating from a small source.
- Define polarization, and explain what is meant by polarized EM radiation — in particular, polarized light.
- Define visible radiation, and determine its limits numerically in terms of wavelength, frequency, and energy.
- Describe the reflection and refraction of EM radiation — especially light — and set up an experiment to verify the two laws.
- Differentiate between alpha and beta radiation and gamma radiation.
- Briefly explain each of the three parts in the symbol ${}_{92}U^{238}$.
- Given the appropriate equipment, illustrate and verify the inverse-square law of EM radiation in the visible region.
- Given the appropriate equipment, produce and detect polarized light in the microwave region.

DISCUSSION

Radiation is the movement of energy along a straight line away from an energy source. The direction of energy flow may be changed after the energy leaves the source; but unless the direction is changed, energy will continue to move radially away from the source. There are three basic types of radiation:

Sound waves are an example of a mechanical vibration. The energy is transmitted through a material medium by pressure variations. This type of radiation is presented in Concept Module 11-0, "Vibrations and Waves" and will not be discussed further here.

Light is an example of electromagnetic radiation, in which energy is transmitted by variations in electric and magnetic fields. Electromagnetic radiation requires no material medium and will travel through a vacuum.

Alpha and beta particles are examples of particle radiation, in which a particle having mass transmits energy in the form of the kinetic energy of its mass.

This module begins with a discussion of simple experimental methods for the creation of radiation. Then the family of EM radiations is examined; this family is neatly arranged according to frequency (or wavelength) in the so-called "electromagnetic spectrum." A study is made of the fundamental similarity of all EM radiation, as well as of the fundamental difference between the individual parts.

Next, a simple picture is presented of just what makes up an electromagnetic wave and how much radiation is propagated in empty space. Following this is (1) a closer look at the origin of visible, infrared, and microwave radiation in the structure of atoms and molecules; and (2) an examination of a few characteristics of radiation — the inverse-square radiation law, polarization reflection, and radiation. Quite apart from radiant energy, particle radiations, such as alpha and beta rays, are presented. Several laboratory experiments that verify some of the characteristics are included.

RADIANT ENERGY (EM RADIATION) - BASICS

The creation and observation of electromagnetic (EM) radiation is not difficult when a simple incandescent light bulb is turned on in a dark room; visible EM radiation (light) immediately floods the room. Electrical energy pulsing through tungsten filaments in the bulb is converted into visible radiation. Invisible thermal radiation (heat) also is produced, since the glass envelope becomes hot. Thus, both light and thermal radiation are created.

Consider another common example. In the process of jumping cables from the terminals of one car battery to another, a large

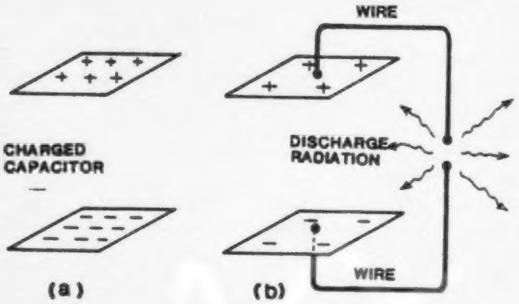


FIG. 1

spark is drawn, especially if the hot wire touches the ground wire. Spark, or discharge flash, is a pulse of radiation energy, partly in the visible and partly in the invisible. Charged capacitors (Figure 1) also produce a spark, or discharge flash, when shorted out, sending a pulse of radiation energy out into the surrounding space. The charged isolated capacitor is shown in (a); the partially discharged capacitor and the EM radiation produced is shown in (b).

Still another common example of EM radiation involves the kitchen toaster. When the toaster is plugged in, the resistive coils increase in temperature. The toaster coils first are observed to be a dull red, then a cherry red, and finally a pinkish-white color. At the beginning of the heating process, the coils radiate primarily thermal energy. The warmth of the wires can be felt, although no color change is observed. Shortly, however, the wires become hotter, change color as described, and then clearly radiate both thermal and light energy.

Since total energy is to be conserved in any system, such as in those described, radiation energy, an integral part, must be conserved. Total work done in a system can be described as follows:

$$\text{Work done on system} = \text{change in kinetic energy} + \text{change in potential energy} + \text{frictional losses} + \text{EM radiation}$$

In mechanical energy systems, no EM radiation is produced. But in electrical energy systems (which involve the movement of electrical charge), EM radiation almost always is produced. For example, immerse a light bulb in a calorimeter and turn the light bulb on for a given period of time T. Carefully measure the temperature of the calorimeter before the light bulb is turned on and again a time T later. The electrical energy supplied to the bulb is given by IVT, where I is the current in amperes, V the voltage in volts, and T the time in seconds. Measurement of the heat energy absorbed by the calorimeter in the same time T, shows that the electrical energy absorbed by the bulb (IVT) does not quite equal the heat energy absorbed by the calorimeter. The input energy is higher! The difference, of course, is equal to the visible, or light, energy which radiated outwards continually while the bulb was on, but was not used in heating the calorimeter. Thus, if energy is to be conserved, as it always must be, all EM radiation must be limited where possible.

THE ELECTROMAGNETIC SPECTRUM

Electromagnetic radiation is a form of energy stored in time-changing electric and magnetic fields. The time-changing electric and magnetic fields always are set into motion by the acceleration of electric charge, whether in a radar or FM transmitting antenna, in a discharging capacitor, or in an a.c. current in a simple light bulb or toaster. Invariably, the exact manner in which the electric charge acceleration occurs always is related to the particular type of electromagnetic radiation that develops. Charge accelerations can be produced in many ways: by varying the motion of charges on an electric conductor (such as on an antenna); by heating a material to increase vibrational energy of charged particles (such as in a tungsten filament); or by causing a charged particle to change its direction (such as in a.c. current).

The different types of EM radiation produced have been identified and organized in accordance with their frequency, or wavelength, range in the so-called EM spectrum. (See Figure 2.) The top scale in Figure 2 identifies frequency from 10 hertz to 10^{22} hertz. The bottom scale identifies the corresponding wavelength from about 10^7 meters down to 10^{-13} meters. Thus, at the left end of the EM spectrum is found the very low-frequency, very long-wavelength radiation, such as a.c. power and radio waves. At the other end is found the very high-frequency, very short-wavelength radiation, which includes the very penetrating x-rays and gamma rays.

The wavelength λ and frequency f for any part of the EM spectrum are related by Equation 1 (the fundamental equation).

$$c = \lambda f \qquad (1)$$

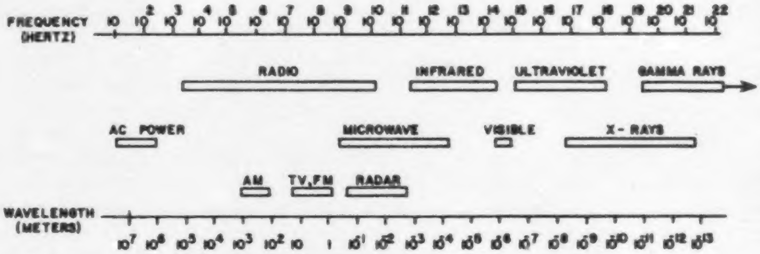

FREQUENCY (HERTZ)
10 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9 10^{10} 10^{11} 10^{12} 10^{13} 10^{14} 10^{15} 10^{16} 10^{17} 10^{18} 10^{19} 10^{20} 10^{21} 10^{22}
RADIO
INFRARED
ULTRAVIOLET
GAMMA RAYS
AC POWER
MICROWAVE
VISIBLE
X-RAYS
AM
TV,FM
RADAR
WAVELENGTH (METERS)
10^7 10^6 10^5 10^4 10^3 10^2 10 1 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5} 10^{-6} 10^{-7} 10^{-8} 10^{-9} 10^{-10} 10^{-11} 10^{-12} 10^{-13}

FIG. 2

where

c = wave speed in meters per second.
λ = wavelength in meters.
f = frequency in hertz (cycles per second).

In free space or vacuum, the speed of EM radiation is a constant equal to 299,792,500 meters per second. (For calculations this number usually is rounded off to 3.0×10^8 m/s.)

The energy associated with electromagnetic radiation is given by Equation 2.

$$E = hf \qquad (2)$$

where

E = energy in joules.

h = famous constant known as Planck's constant and equal to 6.6×10^{-34} J·s.

f = radiation frequency in hertz.

If Equation 1 is used to replace f in Equation 2 by $f = c/\lambda$, then an equivalent form for Equation 2 is shown in Equation 3.

$$E = \frac{hc}{\lambda} \tag{3}$$

Equations 1 and 2 are used to solve problems in Examples A and B, which show that the higher the frequency (or the shorter the wavelength), the higher the energy of the EM radiation. This relationship should be clear from the forms of Equations 2 and 3.

Before leaving the discussion on the EM spectrum, examine Figure 1; and briefly describe some of the more important types of EM radiations.

EXAMPLE A: FREQUENCY AND ENERGY OF VISIBLE RADIATION (LIGHT)

Given: A light has a wavelength of between 400 nanometers (blue) and 700 nanometers (red).

Find: The corresponding frequency and energy of the blue and red light.

Solution: Blue light: (λ = 400 nanometers = 4×10^{-7} m)

From Equation 1: $f_B = \frac{c}{\lambda_B} = \frac{3 \times 10^8}{4 \times 10^{-7}} = 7.5 \times 10^{14}$ hertz

From Equation 2: $E_B = hf_B = (6.6 \times 10^{-34})(7.5 \times 10^{14})$

$E_B = 50 \times 10^{-20}$ joules

Red light: (λ = 700 nanometers = 7×10^{-7} m)

Similarly, $f_R = \frac{c}{\lambda_R} = \frac{3 \times 10^8}{7 \times 10^{-7}} = 4.3 \times 10^{14}$ hertz

$E_R = hf_R = (6.6 \times 10^{-34})(4.3 \times 10^{14})$

$E_R = 28 \times 10^{-20}$ joules

EXAMPLE B: ENERGY COMPARISON

Given: A radio wave has a frequency of 800 kilocycles and a gamma ray of wavelength 10^{-13} meters.

Find: The ratio of their energies.

Solution: Energy of radio wave: use Equation 2, $E = hf$.

$$E_{RW} = 6.6 \times 10^{-34} \times 8 \times 10^{5} = 53 \times 10^{-29} \text{ joules}$$

Energy of gamma ray: use Equation 3, $E = hc/\lambda$.

$$E_G = \frac{(6.6 \times 10^{-34})(3 \times 10^{8})}{10^{-13}} = 20 \times 10^{-13} \text{ joules}$$

Thus,

$$\frac{E_G}{E_{RW}} = \frac{20 \times 10^{-13}}{53 \times 10^{-29}} = 3.8 \times 10^{15}!$$

So the energy of the gamma radiation is about one thousand million million times more energetic than a 800 kilohertz radio wave!

RADIO WAVES

EM radiation with frequencies between 10^4 and 10^7 hertz and wavelengths from tens to thousands of meters are identified as radio waves. They are reflected effectively by charged layers in the upper atmosphere and, therefore, can propagate over long distances from the radio station. The AM frequency band on most table radios varies from 530 kilohertz to 1600 kilohertz.

TELEVISION AND RADAR

Television and FM broadcasting stations make use of EM radiation with frequencies of about 10^8 hertz and wavelengths of about one meter. Most FM dials are calibrated between 85 and 110 megahertz. If the radiated signal is in the form of pulses, the time from emission of the pulse to the reception of its reflected signal (the echo) can be used to determine the distance from source to reflecting object. This is the principle of radar (Radio Detection And Ranging). As Figure 2 indicates, radar is pulsed EM radiation of frequency between 10^9 and 10^{11} hertz and wavelength ranging from a centimeter up to a meter. By means of radar, both the direction and distance of an object can be measured.

INFRARED RADIATION

Radiation with wavelength between 10^{-4} and 10^{-6} meters generally is classified as infrared radiation. It is called "infrared" because most of the radiation is in a region of the spectrum at which the wavelengths are slightly longer than the red end of the visible spectrum. Infrared radiation is emitted by all hot bodies, such as the sun, toasters, room radiators, and soldering irons.

VISIBLE LIGHT

The human eye is sensitive to electromagnetic radiation between 400 and 700 nanometers, and this EM radiation is detected directly with the eye. It has come to be known as "light." The eye reacts most sensitively to the green and yellow colors near 555 nanometers and least sensitively to the far red (where the invisible infrared begins) and to the far blue (where the invisible ultraviolet light begins).

ULTRAVIOLET LIGHT

EM waves shorter than 400 nanometers and longer than ten nanometers (between 10^{15} and 10^{18} hertz) are classified as "ultraviolet" light. Ultraviolet (uv) light, or "black light," as it sometimes is called, is invisible. It is more energetic than either infrared or visible light and causes summer suntans by producing a dark skin pigment known as "melanin." Many atoms emit characteristic uv light during processes of de-excitation. As a result, spectroscopists use the uv "fingerprints" to identify atoms from their characteristic emissions.

X-RAYS

The region of the EM spectrum with radiation of wavelgnths between 10^{-8} and 10^{-17} meters frequently is referred to as "x-radiation" or, more simply, as "x-rays." This radiation is so energetic that it can penetrate tissue and flesh, but not bone, therefore making possible x-ray photographs of the human skeletal frame. X-rays are produced by the sudden deceleration or stopping of electron beams as they strike a metal target. X-rays, with wavelengths roughly comparable to the distance between planes of atoms in crystallographic substances, have been diffracted readily. Such diffraction of x-rays has led crystallographers to important information concerning the structure of crystals and of complex molecules.

GAMMA RAYS

Gamma radiation, very penetrating and very energetic, extends beyond the x-ray region, although some overlapping does exist. Since an x-ray and gamma ray can exist with the same wavelength, frequency, and energy, and are identical, therefore, as EM radiations, how are they distinguished? The answer lies in how they are produced. If the production of EM radiation of wavelength 10^{-17} meters occurs by electron rearrangement in the region of the atom outside the nucleus, the radiation is referred to as x-rays. If the same wavelength radiation is produced by a rearrangement of neutrons and protons in the nucleus, it is referred to as a gamma ray. If EM radiation of 10^{-17} meters is encountered in the laboratory without knowing its origin, it is not known whether it is an x-ray or a gamma ray.

This overlapping of EM radiations (Figure 2) occurs not only for x-radiation and gamma radiation, but also for radio, microwave, and radar. In the overlap region, a decision is required to determine whether it is a radio wave, a microwave, or a radar wave, depending upon how it is produced.

Gamma rays often are discussed in conjunction with alpha and beta rays, which is quite natural, since all three generally are created in nuclear transformations. In fact, the creation of alpha and beta rays in nuclear decay processes almost always is accompanied by the creation of gamma rays. But gamma rays are electromagnetic in character, while alpha and beta rays are particulate in nature. Thus, they fundamentally are different. Particle radiation will be discussed in more depth later in this module.

THE NATURE OF ELECTROMAGNETIC WAVES

Electromagnetic radiation is an electromagnetic wave of specific frequency, moving through space with a speed $c = \lambda f$. The electromagnetic wave itself (Figure 3) consists of propagating electric and magnetic fields E and H, which vary with time as they move along the direction of propagation. The electric field E is shown oscillating in the y-direction and always in the xy plane. The magnetic field H is shown oscillating in the z direction and always in the xz plane, perpendicular to the electric field E. The radiation energy contained in the wave is shown propagating along the positive x-axis. The maximum value that the electric field E attains while oscillating is labeled E_m; that for the magnetic field H is labeled H_m.

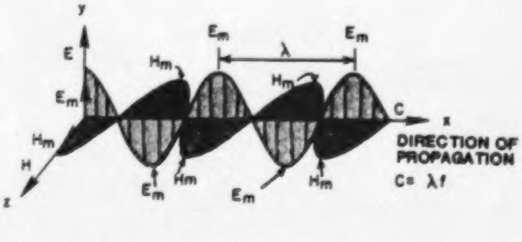


FIG. 3

As explained earlier in Application Module 1E4, an electric field exists at a point in space if a force of electrical origin is exerted on a charged body placed at that point. A force of electrical origin always is produced by the presence of other charged bodies, either positive or negative, and by a magnetic field changing with time. In order to help visualize the electric field around different electric charge configurations or in the vicinity of time-changing

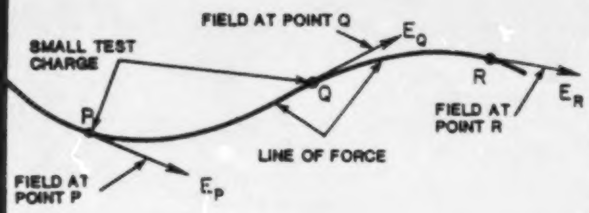
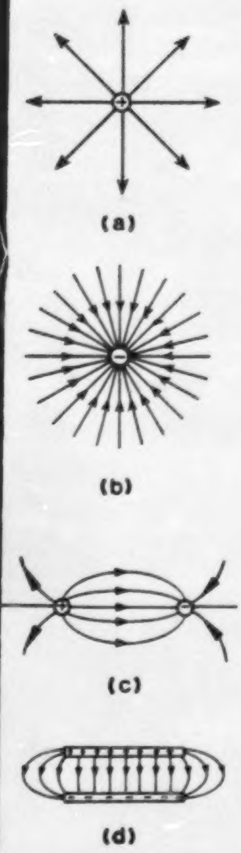


FIG. 4

magnetic fields, the idea of lines of force was introduced in the early 1800s by Michael Faraday, a pioneer in unraveling the mystery of electromagnetic radiation.

An electric line of force is an imaginary line drawn in such a way that its direction at any point — that is, the direction of its tangent — is the same as the direction of the electric force on a very small, positive "test" charge placed at that point. (See Figure 4.) If the number of field lines which are drawn to represent the electric field in a given region are suitably limited, they can be used to indicate visually the magnitude of the field, as well as its direction. Figure 5 shows electric lines of force that surround different configurations of electric charge. The density of lines indicate the relative strength of the electric field. The arrows indicate the direction of the field. Notice that force lines always point in the direction of the net electric force on a small positive "test" charge placed at that point. The "test" charge samples the existing electric field without disturbing or changing it. Figure 5a shows the field surrounding a weak positive charge; the direction of the field is away from the positive charge. Figure 5b shows a stronger negative charge and the electric field it produces. In this case, direction of the field is toward the negative charge. Figures 5c and 5d show the electric field produced by a pair of point charges and the field between two charged parallel plates.


(a)

(b)

(c)

(d)

FIG. 5

Similarly, as presented in Application Module 1E4, a magnetic field exists in regions surrounding ordinary bar magnets, and moves electrical charge as in current-carrying conductors and in the region where an electric field is changing with time. A magnetic field is evidence of a magnetic force, different from the electric force described above. A magnetic force is sensed by a moving charge, in addition to an electric force sensed by the same moving charge.

Magnetic lines of force, like electric lines of force, are used to help visualize a magnetic field. Like the lines of electric force, they are imaginary. The magnetic lines of force at a given position in space always point in the direction of the net magnetic force on a small, imaginary, "free" north pole placed at that point. (Of course, "free," or isolated, north poles do not exist, as do small positive "test" charges. The existence of a north pole always implies a south pole nearby. But these facts do not prevent the direction of force on such a "free north pole" from being deducted if there is one; definitions for the direction of magnetic lines of force remains valid and useful.)

Magnetic lines of force around bar magnets and certain configurations of moving charge are illustrated in Figure 6. The magnetic field surrounding a simple bar magnet is shown in 6a, and Figure 6b illustrates the field produced when the south pole of one magnet is placed near the north pole of another. Figure 6c displays the field of a solenoid with a current flowing in the direction indicated. The right end of the solenoid is a south pole, and the left end is a north pole. The magnetic lines of force in a plane perpendicular to a wire carrying a d.c. current are shown in 6d.

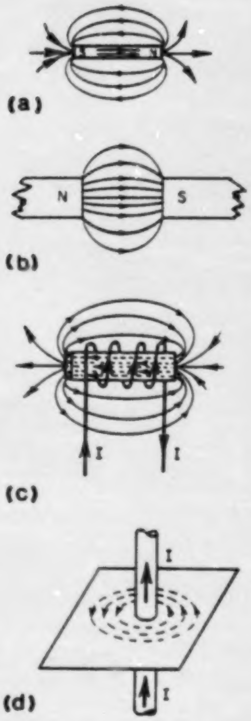


FIG. 6

The representation of electric and magnetic fields as lines of force may be used to illustrate the important principle mentioned earlier; that is, that a time-changing electric field in space produces a magnetic field nearby, and a time-changing magnetic field in space produces an electric field nearby.

Figure 7 illustrates how a changing electric field E developed between two conducting plates produces a magnetic field. In 7a, a static electric field exists in the region between the plates (vertical arrows from bottom plate to top plate). In 7b, the electric field E begins to increase, thereby creating a magnetic field H (concentric circles about axis of plates). In 7c, the electric field continues to grow rapidly, creating an even more intense H field. In 7d, the electric field is still increasing, but more slowly; the H field begins to decrease. In 7e, the electric field has reached its maximum value and is no longer changing; the H field has vanished. In 7f, the electric field begins to decrease; again a magnetic field is produced but now in the opposite direction. In 7g, the electric field is decreasing rapidly, thereby producing a strong H field. In 7h, the electric field decreases more slowly so the magnetic field H begins to weaken. In 7i (same as 7a), the electric field has decreased to its original static value, and the H field has vanished.

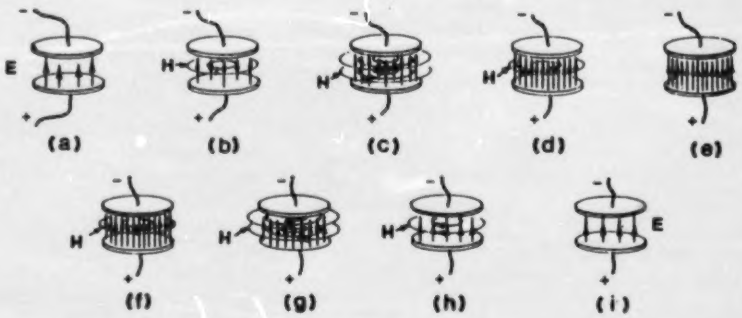


FIG. 7

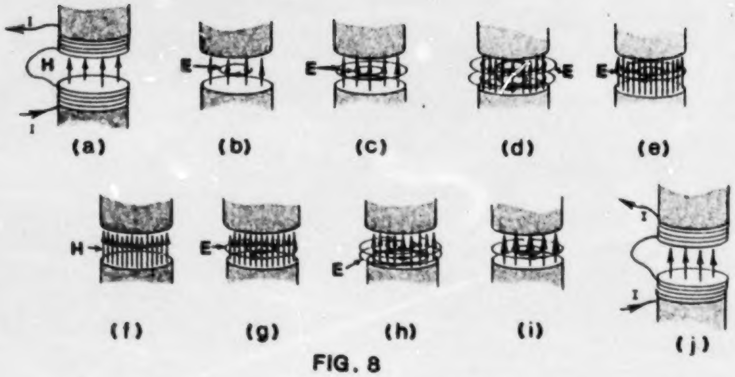


FIG. 8

In Figure 8a, the changing magnetic field H developed between the two poles of the electromagnet in 8a, produces a magnetic field. As the current I through the coils increases, the magnetic field increases in strength (8b, 8c, 8d, and 8e); and an electric field is produced (concentric circles around axis of electromagnet). The strength of the E field is greatest at the instant the magnetic field is increasing the fastest (8d). When the magnetic field has reached its maximum value (8f), the electric field is zero, since the H-field is not changing. Then, as the magnetic field begins decreasing (8g, 8h, 8i) because of a decreasing current in the coils, the electric field again builds to a maximum (8h) and eventually disappears (8j) when the magnetic field returns to its original static value (8a).

The conclusion drawn from Figures 7 and 8 is very important. It provides an explanation of how EM radiation is able to propagate through empty space, quite unlike sound, for example, which requires a medium, or substance, for its transmission. Begin only with a source such as an antenna, where electric charge is accelerated. The physical process of accelerating charge creates a time-changing electric field and magnetic field near the source. Each time-changing field, in turn, creates the other, as shown in Figures 7 and 8, thereby sustaining the progagating electric and magnetic fields (electromagnetic waves or electromagnetic radiation) throughout space. Figure 9 consists of a drawing (similar to Figure 3, except in two dimensions) of a plane EM wave propagating away from a "horn." Internally, electric oscillations in an electronic circuit are led onto an antenna-like

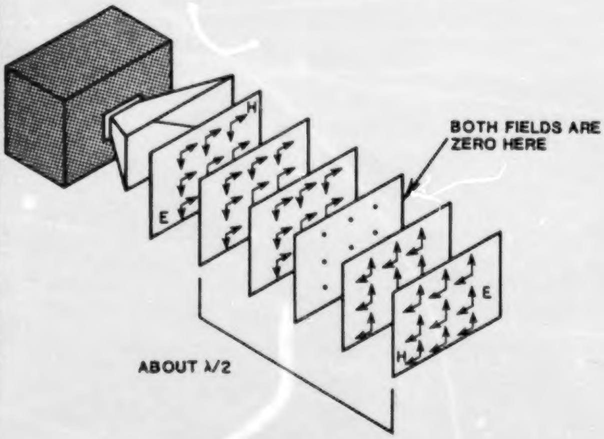


FIG. 9

rod in the conducting horn. There, electric and magnetic fields are generated and radiate out into space, as shown. The H field vibrates from left-to-right, and the E field vibrates up and down. Notice the plane at which both fields go to zero. Notice, also, that when E reverses — goes from "pointing-down" to "pointing-up" — H also reverses, switching from right to left. (Can you justify the note on the drawing about λ/2?)

EM RADIATION - WAVES OR PHOTONS

Inevitably, while learning about electromagnetic energy, one hears EM radiation or energy referred to sometimes as a "wave" and sometimes as a "photon." The idea of waves is very familiar. Therefore, use is made of the concept of waves in describing EM radiation thus far. A photon — similar to a localized EM wave, truncated at both ends (Figure 10) — is not so familiar. Yet the concept of a photon, which is more like the concept of a particle than of a wave, is necessary for one to comprehend EM radiation. EM radiation sometimes is best described as a wave (radar, for example) and at other times best described as a photon (gamma rays, for example).

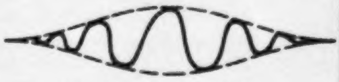


FIG. 10

Light often is referred as a wave phenomena and thought of as matter being made up of particles. Waves are described in terms of amplitude, wavelength, frequency, and energy. Particles, on the other hand, are described in terms of size, mass, momentum, and energy. Visible light waves and radio waves do not exhibit particle properties readily, and particles large enough to be seen with the eye (dust particles, B-B's, baseballs) do not resemble waves. The particle properties of radar and radio-frequency photons are not apparent, because they are of such low energy that only large numbers of them (not single ones) can be detected. When detected in large numbers, radar and radio-frequency photons with comparatively long wavelengths behave exactly like waves. However, the shorter-wavelength, more energetic EM radiations, like energetic and gamma rays, are detectable as "single particles." They do, for example, exhibit true particle properties and are best treated as photons. Not only are waves often described best in terms of particle properties (photons), but very small particles (atoms, electrons, protons) sometimes are described best as waves. These descriptions, by the way, are what quantum mechanics is all about.

The dual nature of EM radiation becomes apparent in the x-ray region, since x-ray photons exhibit both wave and particle properties. Gamma rays, with wavelengths much shorter than x-rays, exhibit primarily particle properties and are treated primarily as photons. In general, EM radiation is treated as a wave when it is of long wavelength (low energy) and when the propagation, reflection, refraction, interference, or diffraction of EM energy is under investigation. EM radiation is treated as a particle (photon) when it is of short wavelength (high energy) and when its interaction with matter (absorption, scattering, transmission) is under investigation.

Before leaving this fascinating topic of waves and photons, a few words about the origin of EM radiation in the visible, infrared, and microwave regions of the EM spectrum are desirable. Most of the visible light seen comes from changes in configurations of electrons within the atom. Thus, when an atom absorbs energy in some form, its electrons pick up this energy and are repositioned further away from the nucleus. When the "excited" atom releases the absorbed energy and, in the process, the electrons are returned to their original configuration, the released energy comes off as short bursts of EM radiation — photons. The frequency of this emitted EM radiation — when it lies between 4.3×10^{14} hertz (red light) and 7.5×10^{14} hertz (blue light) — constitutes the visible part of the spectrum.

In addition to the energy stored in various configurations within the atom and later emitted as light energy, energy also is stored in the vibrational and rotational modes of several atoms locked together in the form of molecules. If the stored molecular energy is of vibrational form (two atoms vibrating back and forth as in the CO molecule) and then released, the EM radiation given off is primarily infrared radiation. Thus, heat energy, which is EM radiation in the infrared region, has its origin in the excess vibrational energy stored in molecules. If the molecular energy is stored as rotational energy (for example, a diatomic molecule rotating around its center of mass like a rigid bar bell) and then released, the EM radiation is given off primarily as microwave radiation.

When studying the processes involved in the emission and absorption of visible, ultraviolet, infrared, or microwave radiation, the concept of short bursts of energy (photons), rather than of long unending wave trains, best describes what is taking place. Thus, the phenomena of absorption and emission of EM radiation are described and most easily understood in terms of wave packets or photons of EM energy.

SOME CHARACTERISTICS OF EM RADIATION

Since EM radiation is fundamentally an electromagnetic wave, it exhibits all the behaviors expected of waves propagating through space and passing through and around objects. Thus, EM radiation is expected to be reflected and refracted; to diminish in strength, or attenuate, as it spreads out from a source; to be polarized in some instances; to show interference when it overlaps itself; and to be diffracted when it passes through small portholes or around small objects. Here "small" refers to a dimension of the order of the wavelength of the EM radiation in question. Clearly, "small" refers to two entirely different dimensions with reference to radio waves and x-rays.

REFLECTION

The law of reflection of EM waves is illustrated in Figure 11. The ray shown in Figure 11 can be thought of as the x-axis (or direction of propagation) of the E and H fields shown in Figure 3. Notice that in Figure 11, the incident ray, reflected ray, and normal all lie in the same plane and define the plane of incidence. Angle A, the angle of incidence, is equal to angle B, the angle of reflection. The law of reflection of EM waves is given by Equation 4.

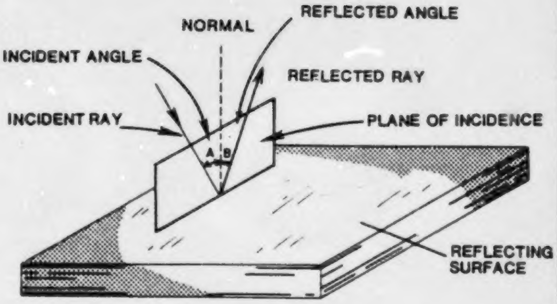

REFLECTED ANGLE
NORMAL
INCIDENT ANGLE
REFLECTED RAY
INCIDENT RAY
PLANE OF INCIDENCE
A
B
REFLECTING SURFACE

FIG. 11

$$\angle A = \angle B \qquad (4)$$

where

$\angle A$ = angle reflection.
$\angle B$ = angle of incidence.

REFRACTION

The law of refraction of EM radiation is given by Equation 5 and illustrated in Figure 12. Again,

$$n_a \sin A = n_c \sin C \qquad (5)$$

where

n_a = index of refraction of incident ray.
n_c = index of refraction of ray in material.

Use is made of rays that indicate the direction along which the EM radiation propagates. The index of refraction n is a constant characteristic of the material and is merely the ratio of the speed of light c in a vacuum to the speed of light v in that material. For air, the index of refraction n is nearly 1.00. For lucite and most glasses, it is around 1.50.

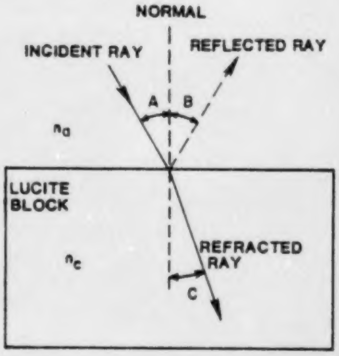

NORMAL
INCIDENT RAY
REFLECTED RAY
A
B
n_a
LUCITE BLOCK
n_c
REFRACTED RAY
C

FIG. 12

POLARIZATION

An examination of Figures 3 and 9 presented earlier discloses two facts. First, the vibration of the electric (E) and magnetic (H) fields always is perpendicular to the direction of progagation of the EM wave. This has come to be recognized as the transverse nature of EM radiation. Second, in Figure 3 the E-vibration always is in the y-direction, and the H-vibration is always in the z-direction. Such a restriction of vibration of E or H along a single direction is called "polarization." Thus, the electric field is said to be polarized along the y-direction, and the magnetic field to be along the z-direction. The direction of propagation is along the x-direction. Similarly, in Figure 9 the EM radiation is polarized linearly; the H-field is polarized left-to-right; and the E-field is polarized up and down.

INVERSE-SQUARE LAW

If a person stands some distance from a 100-watt lamp bulb (far enough away that the distance from the bulb is large compared to the size of the bulb), the bulb can be observed and treated as if it were a "point" source of EM radiation. The power radiated by the bulb decreases as the square of the distance between the receiver and the bulb. Thus, if the power per unit area is measured to be P_o when 10 feet away, the power per unit area will be $P_o/4$ when 20 feet away and $P_o/9$ when 30 feet away, and so on. This rapid decrease in power with doubled and tripled separations is known as the inverse-square law and is given in simple form by Equation 6:

$$P = \frac{P_o}{d^2} \qquad (6)$$

where

P = power per unit area falling normally on a collecting surface a distance d from the light source.

P_o = power per unit area falling normally on a collecting surface a distance d of unit length from the light source.

PARTICLE RADIATION

As previously stated in this module, "particle radiation" is very different from "electromagnetic radiation," although both use the genertic term "radiation." Gamma rays (an electromagnetic radiation) differ significantly from alpha and beta rays (particle radiation), although all are created during nuclear transformations and, therefore, usually discussed together. In order that the difference between electromagnetic radiation and particle radiation can be clarified, the origin and characteristics of alpha and beta radiation must be examined.

Recall that atoms are composed of a positive nucleus surrounded by negative electrons. The nucleus contains both protons and neutrons. Protons have a mass equal to that of 1836 electrons and have an electronic charge of +1. (Electrons have a charge of -1.) Neutrons have a slightly greater mass than protons and have a charge of zero. Under certain conditions, a neutron in an atomic nucleus may divide into a proton (+1) and an electron (-1).

Beta radiation, or beta rays, is a stream of very energetic electrons emitted from the nucleus during the instant at which neutrons are converted to protons within the nucleus, therefore, beta rays really are electrons born in the nucleus! Alpha rays are also particles — the nuclei of helium atoms, in fact, and about 7500 times heavier than the electrons in beta rays. The alpha particles, like beta particles, also are created in the nucleus and emitted during a nuclear transformation. Often, alpha, beta and gamma radiation are referred to as a group; however, alpha and beta rays refer to streams of energetic particles, and gamma rays, like x-rays, refer to energetic electromagnetic radiation. Both x-rays and gamma rays have particle-like properties and often are treated as photons rather than as waves, but they remain fundamentally electromagentic waves and not particles.

ALPHA RAYS

As mentioned above, the alpha particle is the nucleus of a helium atom expelled from a heavy element during the nuclear transformation from one element to another. Thus, for example, when uranium-238, which has 92 protons and 146 neutrons clustered together in its nucleus, changes, or "decays," to thorium-234 (90 protons and 144 neutrons), an alpha particle is emitted from the nucleus. This alpha particle (uranium-238) has two protons and two neutrons, just as does the fully-ionized helium atom. In terms of common nuclear symbols, the nuclear reaction discussed here is given as follows:

$${}_{92}U^{238} \rightarrow {}_{90}Th^{234} + {}_{2}He^{4}$$

Notice that superscripts (mass numbers) and subscripts (atomic numbers) obey a conservation law; thus, $238 = 234 + 4$, and $92 = 90 + 2$. The element uranium-238 is written as ${}_{92}U^{238}$, where the subscript 92 refers to the number of protons in the nucleus (atomic number), and the superscript 238 refers to the sum of protons (92) and neutrons (146) in the nucleus (atomic mass numbers). The reaction equation shows that an alpha particle ${}_{2}He^{4}$ is created when uranium-238 decays to thorium-234.

Alpha particles usually are very energetic, but are able to travel only short distances in matter because of their excessive mass. (As mentioned above, the alpha particle is about 7500 times as massive as the electron.) Although most alpha particles are created with energies from 4 million electron volts to 9 million electron volts (with corresponding velocities around 1.4×10^7 m/s to 2.1×10^7 m/s), almost all alpha particles can be stopped by a single sheet of paper. (Note: one electron volt is the energy acquired by an electron accelerated through a potential difference of one volt.) The range of alpha rays in air can be remembered by use of a general rule of thumb — about 1 cm of air for each 2 million electron volts of energy. Even in air, therefore, the most energetic alpha particles will not travel more than about three or four inches.

BETA RAYS

Electrons emitted by radioactive nuclei are called "beta particles" (or beta rays) — a name given to them before they were identified as electrons. So beta rays, as mentioned above, are nothing more than high-energy electrons created in the nucleus. Since, so far as known, there are no electrons as such in the nucleus, they must be created during the nuclear transformation itself. This process is explained in the following manner: a neutron (which does exist in the nucleus) is converted to a proton (which also can exist in the nucleus) by creating an electron. In terms of a nuclear reaction equation, this relationship is written —

$$ {}_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0. $$

In essence, the equation states that a neutron with charge zero and mass number one (written ${}_0n^1$) is converted to a proton of charge one and mass number one (written ${}_1p^1$) and an electron of charge minus one and mass number zero (written ${}_{-1}e^0$).

The process described above occurs in many nuclear transformations, thereby giving rise to beta rays. An example of such a transformation is —

$$ {}_{82}Pb^{214} \rightarrow {}_{83}Bi^{214} + {}_{-1}e^0. $$

Radioactive lead-214 decays to the isotope bismuth-214, and in the process emits a beta particle (a high-speed electron). Since a neutron was converted into a proton within the nucleus, the mass number 214 (sum of protons and neutrons) remains the same before and after the transformation. But the atomic numbers (number of protons) increases by one, going from 82 to 83, because a neutron in the nucleus was changed to a proton.

The penetration of matter by the beta particles generally is much higher than that by alpha particles. A single sheet of paper stops most alpha particles; a ream of paper (or several centimeters of aluminum) is required to stop beta particles. The range of energy values for beta particles is not defined as clearly as for alpha particles. In fact, beta rays have a continuous energy distribution, with the energy of a typical beta particle lying around a few hundred thousand electron volts. However, the velocity of the beta particles is about 2.7×10^8 m/s (nine-tenths the speed of light), considerably higher than that of the alpha particle.

The beta particle must move with a much higher speed than an alpha particle of the same energy, since the beta particle is so much lighter in mass. Beta particles move deeper into matter than alpha particles, because of their higher speeds and lesser net charge.

Table 1 summarizes the important characteristics of alpha and beta particles.

TABLE 1: CHARACTERISTICS OF α AND β PARTICLES

	Symbol	Mass	Charge	Approximate Energy	Range
Alpha Particle	α, $_2He^4$	6.69×10^{-27} kg	3.2×10^{-19} C	4-9 MeV	Stopped by one sheet of paper
Beta Particle	β, $_{-1}e^0$	9.1×10^{-31} kg	1.6×10^{-19} C	1 MeV	Stopped by one ream of paper

REFLECTION AND REFRACTION OF EM RADIATION

MATERIALS

Low-power He-Ne laser (1-3 milliwatts).
Optical material with plane parallel surfaces and an index of refraction of about 1.5 (a large block of lucite or acrylic plastic works well).
Front-surface plane mirror.
Drafting paper, protractor, and sharp pencils.

PROCEDURE

In this experiment, the laws of reflection and refraction with EM radiation (laser light at 6328 Å in the visible region of the EM spectrum) will be verified. Observe safety precautions when using laser light! Never look into the direct beam or permit the reflected beam to enter the eyes!!

REFLECTION

1. Set up experimental apparatus as shown in Figure 13.
2. Place center of front-surface plane mirror flush with base of protractor at center of rotating mount.
3. Raise or lower rotating mount so that laser beam just grazes protractor surface and strikes mirror at the center of mount. Light is then reflected straight back towards the laser. Rotate mount to five different settings (10°, 20°, 30°, 40°, 50°) and record angle of incidence and angle of reflection in Data Table. (If laser beam grazes top surface of the protractor properly, both angle of incidence and angle of reflection can be read directly off the protractor.) Verify law of reflection.

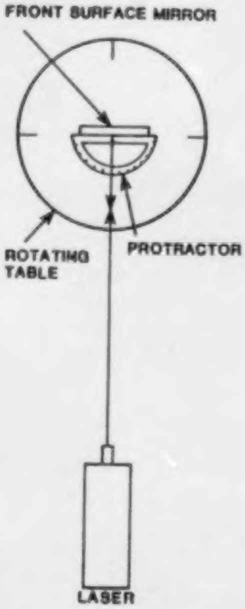


FIG. 13

REFRACTION

1. Draw a straight line NN on a large piece of drafting paper, as shown in Figure 14.

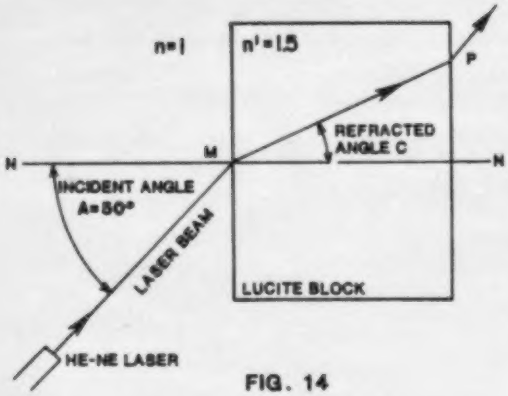


FIG. 14

2. Align lucite block in such a manner that one of its parallel surfaces is perpendicular to line NN at M.

3. Orient low-power He-Ne laser in such a manner that laser beam is incident on lucite block at point M and makes an angle of 50° with line NM. Clearly mark point M.

4. Locate and mark P where laser beam emerges from lucite block on opposite side. Now remove block and draw line MP.

5. Measure refracted angle C with protractor.

6. Now calculate angle C from law of refraction ($n_a \sin A = n_c \sin C$). Remember that $n_a = 1$, $n_c = 1.5$ and angle A = 50°.

7. Compare calculated value of C in step 6 with measured value in step 5. Account for any difference.

8. The laser beam leaving the lucite block at point P should be parallel to the incident laser beam (50°) at M: Draw the exiting ray, and repeat steps 5, 6, and 7 for point P, taking care to redefine the angles of incidence and refraction properly for point P.

POLARIZATION OF EM RADIATION

MATERIALS

Laboratory microwave transmitter (wavelength around 3 cm).
Laboratory microwave receiver or detector.
Microwave grid analyzer (a series of parallel wires with spaces of about 1/10 wavelength).

(Note: The entire microwave demonstration kit is available from CENCO or WELCH laboratory apparatus companies.)

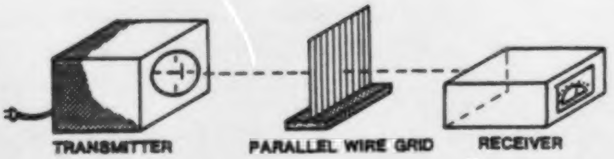


FIG. 15

PROCEDURE

EM radiation emitted by the mircrowave transmitter is polarized. Arrange transmitter, grid, and receiver as shown in Figure 15.

1. Remove grid, turn on transmitter, and move receiver (also adjust sensitivity) to record direct beam at near full scale.

2. Replace grid, and slowly rotate it from an orientation with wires vertical, as shown in Figure 15, to an orientation with wires horizontal. Observe receiver reading carefully as grid is rotated.

3. At what angle of rotation of grid (orientation of wires measured from vertical) did receiver read its minimum value? (Repeat step 2 until same reading can be reproduced several times without trouble.)

4. From observation in step 3, determine direction of polarization of the E and H field in microwave radiation. (Consult instructor if necessary.) Once certain of answer, draw a representation of EM field similar to that shown in Figure 9, with correct E and H polarizations. Draw sufficient planes to cover one wavelength of variation in E and H.

INVERSE-SQUARE LAW

MATERIALS

Photographic light meter.

PROCEDURE

In a room in which all light can be extinguished, arrange a test lamp and photographic light meter on a long table, as shown in Figure 16.

1. Locate photographic light meter at a distance $d = D_1$ from a frosted incandescent lamp (choose the 100-watt bulb initially) in such a manner that light meter registers nearly full scale when lamp is on and all other room lights are turned off. Ensure that front surface of the light meter is perpendicular to line joining center of lamp and center of light meter (line AB in Figure 14). Record initial reading P_1.

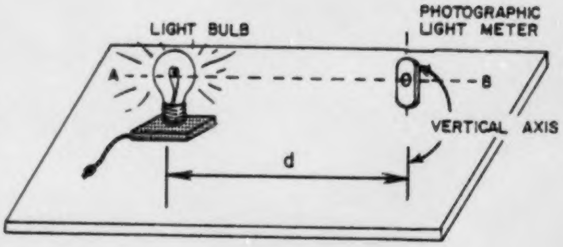


FIG. 16

2. Carefully move light meter away from lamp along line AB; and take successive light-meter readings at 1.5 D_1, 2 D_1 and 3 D_1. Ensure that face of light meter is normal to line AB when taking each reading. (Rotate light meter back and forth about its vertical axis, as shown in Figure 15, until highest reading is recorded. Face of light meter then should be normal to the line AB.)

3. Using Equation 6 ($P = P_o/d^2$), verify the inverse-square law by showing that P_2 (at $d = 1.5\ D_1$) is equal to (1/2.25) P_1; that P_3 (at $d = 2\ D_1$) is equal to (1/4) P_1; and that P_4 (at $d = 3\ D_1$) is equal to (1/9) P_1.

4. Repeat the entire procedure for 40-watt bulb. How well did the results verify the inverse-square law dependence of power per unit area on distance?

   Note: Possible errors include the following:

   1. Light meter too close to lamp. Recall that the law is valid only for point sources and that the bulb is not a point source. The distance D_1 should be at least ten times the diameter of the bulb. Results for the 100-watt lamp should be more valid than for the 40-watt lamp, since the light meter for the 100-watt lamp will be further away for the initial full-scale reading.

   2. Room light, other than from bulb striking the light meter.

   3. Surface of light meter not perpendicular to line AB at time of readings.

   4. Illuminance scale on light meter too difficult to read accurately.

SELF CHECK

1. Discuss the basic properties of these types of radiation:

   Sound.
   Light.
   Alpha and beta particles.

2. What is electromagnetic radiation (radiant energy)? Describe a simple experiment that demonstrates how electromagnetic radiation can be created.

3. The microwave demonstration transmitter sold by the Welch

Scientific Company emits microwaves of 3 cm wavelength. Determine the frequency (hertz) and energy (joules) of the microwaves.

4. The eye is most sensitive to visible light at 555 nanometers. What is the frequency of this light? What is its color? What is its energy?

5. A 150-watt incandescent lamp hangs 8 feet above the surface of a study desk. How close to the desk must it be moved to increase the power per unit area falling on the surface by a factor of six?

6. From Figure 2, the EM spectrum, determine the midpoint frequency for each EM radiation shown (from a.c. power to gamma rays), and calculate the corresponding wavelength and EM wave energy. Present results in the Data Table.

7. What is the difference between a low energy gamma ray of frequency 10^{20} hertz and a high-energy x-ray of the same frequency? Explain.

8. A gamma ray photon has an energy of 3 million electron volts. (One electron-volt of energy is equal to 1.6×10^{-19} joules.) What is the frequency and wavelength of this photon? What is its speed in glass ($n_{glass} = 1.5$)?

DATA TABLE

EM Radiation	Mid-Frequency (From Figure 2)	Wavelength (Calculated)	Energy (Joules) Calculated
a.c. power			
Radio			
AM			
TV, FM			
Radar			
Microwave			
Infrared			
Visible			
Ultraviolet			
X-rays			
Gamma rays			

9. An experiment is performed to determine a numerical value for Planck's constant. In the experiment it is found that an EM wave of frequency 1×10^{11} Hz carries an energy of 6.8×10^{-23} J. From this data point, find Planck's constant and determine its percent difference from the accepted value.

10. What should be the slope of a line which is obtained by plotting energy versus frequency?

11. What is the index of refraction of a plastic if light incident on the plastic at an angle of 30° (measured from the normal), makes an angle of 20° with the normal inside the plastic? Assume the medium surrounding the plastic is air.

12. Light is incident on a mirror at an angle of 90° with the plane of the mirror. If the mirror is now rotated by an angle θ, what angle does the reflected light now make with the incident light?

13. If 600 watts of power fall on an area of 3 square meters when the light source is 6 meters away how much power falls on the same area when the light is brought to 3 meters away?

14. What is the index of refraction of a material if light travels 2.6×10^{8} m/s in this material?

15. The frequency of a light wave is independent of the medium in which it is traveling. What is the wavelength of a 10^{10} Hz light wave in glass.

16. A diver looks up at the sun from his position at the bottom of a lake. It appears to him that the sun is at an angle of 25° from zenith (directly overhead). What angle do the sun's ray make with the surface of the lake if the index of refraction of water is 1.33?

17. How far does light travel in air in one year?

18. How long does it take light to reach us from a star 20×10^{15} miles away?

19. Which has a longer wavelength, yellow light in air or red light in glass?

20. A laser beam can deliver 4000 calories/s. How long would it take this laser beam to raise the temperature of 300 grams of water 10C°?

21. Light makes an angle of 60° with the normal in a glass plate. Use Snell's law to try to determine the angle at which this light will emerge into the air. Is the result reasonable? Why?

22. The ratio of the frequencies of two light waves is 6/8. What is the ratio of their wavelengths in air?

23. Quite often Equation 3 is written as E = tr w, where w is called the angluar frequency and is equal to 2 πf. What is the relationship between tr and h?

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APPENDIX

PHYSICAL CONSTANTS AND CONVERSION FACTORS

Section 1: Greek Alphabet

Section 2: Selected Physical Constants

Section 3: Selected Astronomical Data

Section 4: Tables of Conversion Factors

SECTION 1: GREEK ALPHABET

NAME	UC	COMMONLY DESIGNATES	LC	COMMONLY DESIGNATES
Alpha	Α		α	Angles, area, absorption factor, atten. constant, I gain CB confg.
Beta	Β		β	Angles, coefficients, phase constant, flux density, I gain CE confg.
Gamma	Γ	Complex propagation constant	γ	Angles, specific gravity, elect. conductivity, propag'n constant
Delta	Δ	Increment, determinant, permittivity, variation	δ	Angles, density, increment
Epsilon	Ε		ϵ	Base of natural logs, dielectric constant, electrical intensity
Zeta	Ζ	Impedance	ζ	Coordinates, coefficients
Eta	Η		η	Hysteresis, coordinates, efficiency intrinsic impedance
Theta	Θ		θ	Angular phase displacement, time constant, reluctance
Iota	Ι	Current	ι	Unit vector
Kappa	Κ		κ	Coupling coefficient, susceptibility, dielectric constant
Lambda	Λ	Permeance	λ	Wavelength, attenuation constant
Mu	Μ		μ	Prefix micro, amplification factor, Permeability
Nu	Ν		ν	Frequency, reluctivity
Xi	Ξ		ξ	Coordinates, output coefficients
Omicron	Ο		ο	Reference point
Pi	Π		π	3.1416
Rho	Ρ		ρ	Resistivity, volume charge density, coordinates
Sigma	Σ	Summation	σ	Electrical conductivity, leakage coefficient, complex propag'n constant
Tau	Τ		τ	Time constant, time phase displacement, transmission factor
Upsilon	Υ		υ	
Phi	Φ	Scalar potential, magnetic flux, radiant flux	ϕ	Phase angle
Chi	Χ		χ	Angles, electrical susceptibility
Psi	Ψ		ψ	Angles, coordinates, dielectric flux, phase difference
Omega	Ω	Resistance in ohms	ω	Angular velocity ($2\pi f$)

SECTION 2: SELECTED PHYSICAL CONSTANTS

Standard gravitational acceleration (g)	9.807 m/s^2
Standard atmosphere (atm)	101325 N/m^2
Thermochemical kilocalorie	4184 J
Speed of light in a vacuum (c)	2.9979×10^8 m/s $\simeq 3 \times 10^8$ m/s
Electronic charge (e)	1.602×10^{-19} c
Planck constant (h)	6.625×10^{-34} J·s or 4.135×10^{-15} eV·s
Universal gas constant (R)	$8314 \frac{J}{kmol \cdot K}$ or $1.987 \frac{kcal}{kmol \cdot K}$
Volume/kilomole of ideal gas at 1 atm and 0° C	22.4 m^3/kmol
Boltzmann constant (k)	1.381×10^{-23} J/K
Gravitation constant (G)	6.67×10^{-11} $N \cdot m^2/kg^2$
Rest energy of one atomic mass unit	931.48 MeV
Electron volt (eV)	1.602×10^{-19} J

Rest masses of particles:

Electron	9.109×10^{-31} kg	0.511 MeV
Proton	1.673×10^{-27} kg	938.26 MeV
Neutron	1.675×10^{-27} kg	939.55 MeV
Deuteron	3.343×10^{-27} kg	1875.58 MeV
α-particle	6.644×10^{-27} kg	3727.3 MeV

SECTION 3: SELECTED ASTRONOMICAL DATA

Earth

Radius: mean	6371 km = 3959 mi
Distance from sun: mean	149.5×10^6 km = 92.9×10^6 mi
Period of rotation	86,164 s = 1 sidereal day = 23.94 h
Radiation from sun at earth's mean distance	1.35 kW/m^2

Moon

Radius	1,741 km = 1,082 mi
Distance from earth: mean	384,400 km = 239,000 mi
Period of revolution = period of rotation	27.322 d
Mass	7.343×10^{22} kg
Mean density	3.33 Mg/m^3

Sun

Radius	696,500 km = 432,200 mi
Mass	1.987×10^{30} kg
Mean density	1.41 Mg/m^3

SECTION 4: TABLES OF CONVERSION FACTORS

Angle	degree	minute	second	rad	rev
1 degree =	1	60	3600	1.745×10^{-2}	2.778×10^{-3}
1 minute =	1.667×10^{-2}	1	60	2.909×10^{-4}	4.630×10^{-5}
1 second =	2.778×10^{-4}	1.667×10^{-2}	1	4.848×10^{-6}	7.716×10^{-7}
1 radian =	57.30	3438	2.063×10^{5}	1	0.1592
1 revolution =	360	2.16×10^{4}	1.296×10^{6}	6.283	1

Length	m	km	in	ft	mi
1 meter =	1	10^{-3}	39.37	3.281	6.214×10^{-4}
1 kilometer =	1000	1	3.937×10^{4}	3281	0.6214
1 inch =	0.0254	2.54×10^{-5}	1	0.0833	1.578×10^{-5}
1 foot =	0.3048	3.048×10^{-4}	12	1	1.894×10^{-4}
1 statute mile =	1609	1.609	6.336×10^{4}	5280	1

1 angstrom = 10^{-10} m

1 micron (μ) = 1 μm

1 nautical mile = 1852 m = 1.1508 mi = 6076.10 f

1 fathom = 6 ft

1 yard (yd) = 3 ft

1 rod = 16.5 ft

1 mil = 10^{-3} in

1 league = 3 naut miles

Area	m^2	cm^2	ft^2	in^2
1 square mile =	1	10^{4}	10.76	1550
1 square centimeter =	10^{-4}	1	1.076×10^{-3}	0.1550
1 square foot =	9.290×10^{-2}	929.0	1	144
1 square inch =	6.452×10^{-4}	6.452	6.944×10^{-3}	1

1 square mile = 27,878,400 f^2 = 640 acre

1 circular mil = 7.854×10^{-2} in^2

1 acre = 43,560 ft^2

1 barn = 10^{-28} m^2

1 hectare = 10,000 m^2 = 2.471 acre

Volume	m^3	cm^3	ft^3	in^3
1 cubic meter =	1	10^6	35.31	6.102×10^4
1 cubic centimeter =	10^{-6}	1	3.531×10^{-5}	0.06102
1 cubic foot =	2.832×10^{-2}	28.320	1	1728
1 cubic inch =	1.639×10^{-5}	16.39	5.787×10^{-4}	1

1 U.S. fluid gallon = 4 quarts = 8 pints = 128 fluid ounces = 231 in^3

1 British Imperial gallon = the volume of 10 lb of water at 62°F = 277.42 in^3

1 liter = 1000 cm^3

Force	dyne	kgf	N	P	pdl
1 dyne =	1	1.020×10^{-6}	10^{-5}	2.248×10^{-6}	7.233×10^{-5}
1 kilogram-force =	9.807×10^5	1	9.807	2.205	70.93
1 newton =	10^5	0.1020	1	0.2248	7.233
1 pound =	4.448×10^5	0.4536	4.448	1	32.17
1 poundal =	1.383×10^4	1.410×10^{-2}	0.1383	3.108×10^{-2}	1

1 kgf = 9.806 N

1 p = 32.173 pdl

Pressure	atm	inch of water	cm Hg	N/m^2	p/in^2
1 atmosphere =	1	406.8	76	1.013×10^5	14.70
1 inch of water(a) =	2.458×10^{-3}	1	0.1868	249.1	0.03613
1 cm mercury(a) =	1.316×10^{-2}	5.353	1	1333	0.1934
1 newton per m^2 =	9.869×10^{-6}	4.105×10^{-3}	7.501×10^{-4}	1	1.450×10^{-4}
1 pound per i^2 =	6.805×10^{-2}	27.68	5.171	6.895×10^3	1

(a) Under standard acceleration, and temperature of 4°C for water, 0°C for mercury.

1 bar = 10^5 N/m^2

1 torr = 1 mm Hg

1 cm of water = 98.07 N/m^2

1 ft of water = 62.43 p/f^2

Energy	Btu	fp	J	kcal	kWh
1 British thermal unit =	1	777.9	1055	0.2520	2.930×10^{-4}
1 foot-pound =	1.285×10^{-3}	1	1.356	3.240×10^{-4}	3.766×10^{-7}
1 joule =	9.481×10^{-4}	0.7376	1	2.390×10^{-4}	2.778×10^{-7}
1 kilocalorie =	3.968	3086	4184	1	1.163×10^{-3}
1 kilowatt-hour =	3413	2.655×10^{6}	3.6×10^{6}	860.2	1

Power	Btu/h	fp/s	hp	kcal/s	kW	W
1 Btu/h =	1	0.2161	3.929×10^{-4}	7.000×10^{-5}	2.930×10^{-4}	0.2930
1 fp/s =	4.628	1	1.818×10^{-3}	3.239×10^{-4}	1.356×10^{-3}	1.356
1 horsepower =	2545	550	1	0.1782	0.7457	745.7
1 kcal/s =	1.429×10^{4}	3087	5.613	1	4.184	4184
1 kilowatt =	3413	737.6	1.341	0.2390	1	1000
1 watt =	3.413	0.7376	1.341×10^{-3}	2.390×10^{-4}	0.001	1

1 ton (refrigeration) = 12,000 Btu/h

Mass	g	kg	lb	sl	ton
1 gram =	1	0.001	0.002205	6.852×10^{-5}	1.102×10^{-6}
1 kilogram =	1000	1	2.205	6.852×10^{-2}	1.102×10^{-3}
1 pound (avoirdupois) =	453.6	0.4536	1	3.108×10^{-2}	0.0005
1 slug =	1.459×10^{4}	14.59	32.17	1	1.609×10^{-2}
1 ton-mass =	9.072×10^{5}	907.2	2000	62.16	1

1 avoirdupois pound = 16 avoirdupois ounces = 7000 grains = 0.45359237 kg

1 long ton = 2240 lb = 20 cwt

1 stone = 14 lb

1 hundred weight (cwt) = 112 lb

1 metric ton = 1000 kg = 2205 lb

1 carat = 0.2 g

1 pennyweight (dwt) = 24 grains

Time	y	d	h	min	s
1 year =	1	365.2	8.766×10^{3}	5.259×10^{5}	3.156×10^{7}
1 day =	2.738×10^{-3}	1	24	1440	86,400
1 hour =	1.141×10^{-4}	4.167×10^{-2}	1	60	3600
1 minute =	1.901×10^{-6}	6.944×10^{-4}	1.667×10^{-2}	1	60
1 second =	3.169×10^{-8}	1.157×10^{-5}	2.778×10^{-4}	1.667×10^{-2}	1

1 day = period of rotation of earth = 86,164 s

1 year = period of revolution of earth = 365.242 d

Density	sl/f^3	lb/ft^3	lb/in^3	kg/m^3	g/cm^3
1 slug per ft^3 =	1	32.17	1.862×10^{-2}	515.4	0.5154
1 pound per ft^3 =	3.108×10^{-2}	1	5.787×10^{-4}	16.02	1.602×10^{-2}
1 pound per in^3 =	53.71	1728	1	2.768×10^{4}	27.68
1 kg per m^3 =	1.940×10^{-3}	6.243×10^{-2}	3.613×10^{-5}	1	0.0001
1 gram per cm^3 =	1.940	62.43	3.613×10^{-2}	1000	1

Speed	ft/s	km/h	m/s	mi/h	knot
1 foot per second =	1	1.097	0.3048	0.6818	0.5925
1 kilometer per hour =	0.9113	1	0.2778	0.6214	0.5400
1 meter per second =	3.281	3.6	1	2.237	1.944
1 mile per hour =	1.467	1.609	0.4470	1	0.8689
1 knot =	1.688	1.852	0.5144	1.151	1

1 knot = 1 nautical mile/hr

Mass-Energy equivalents	kg	J	MeV
1 kilogram	1	8.987×10^{-16}	5.610×10^{29}
1 nuclidic mass unit	1.6660×10^{-27}	1.492×10^{-10}	931.5
1 joule	1.113×10^{-17}	1	6.242×10^{12}
1 million electron-volts	1.783×10^{-30}	1.602×10^{-13}	1

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